Poisson Matting
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Recall: Matting

• Matting equation over image

\[ I = \alpha F + (1 - \alpha)B \]

- \( F \) – foreground
- \( B \) – background
- \( \alpha \) – alpha matte
Recall: Matting

• Matting equation over image
  \[ I = \alpha F + (1 - \alpha)B \]

• Under-constrained equation:
  – 7 unknown variables
    • \( \alpha \)
    • R,G,B channels of \( F \)
    • R,G,B channels of \( B \)
  – 3 known variables
    • R,G,B channels of \( I \)

• Additional information
  – User-defined Trimap
Poisson Matting

Another approach to solving the matting problem

**Approach:**
- Use gradient of image to calculate gradient of matte
- Reconstruct matte from gradient of matte

**Key addition:**
- Enables user interaction to locally improve results
Poisson Matting

• Matting equation
  \[ I = \alpha F + (1 - \alpha)B \]

• Gradient of matting equation
  \[ \nabla I = (F - B)\nabla \alpha + \alpha \nabla F + (1 - \alpha)\nabla B \]

• Gradient rewritten
  \[ \nabla \alpha = \frac{1}{F - B} (\nabla I - \alpha \nabla F - (1 - \alpha)\nabla B) \]
  \[ A = \frac{1}{F - B} \]
  \[ D = \alpha \nabla F - (1 - \alpha)\nabla B \]

• Once gradient of \( \alpha \) is obtained, we can create matte
  \[ g = -\nabla \alpha \]
  \[ \Delta \alpha = \text{div}(g) \]
Poisson Matting

- Matting equation
  \[ I = \alpha F + (1 - \alpha)B \]

- Gradient of matting equation
  \[ \nabla I = (F - B)\nabla \alpha + \alpha \nabla F + (1 - \alpha)\nabla B \]

- Gradient rewritten
  \[ \nabla \alpha = A(\nabla I - D) \]
  \[ A = \frac{1}{F - B} \]
  \[ D = \alpha \nabla F - (1 - \alpha)\nabla B \]

- Once gradient of \( \alpha \) is obtained, we can create matte
Poisson Matting: Two steps

\[ \nabla \alpha = A(\nabla I - D) \]

- **Step 1:** Global (required)
  - Solve global Poisson matting problem, assumes \( D = 0 \)
- **Step 2:** Local (optional)
  - Adjust local regions with local Poisson matting operations
Step 1: Global Poisson Matting

Global solution assumes $D = 0$ in gradient equation

General Case

$$\nabla \alpha = A (\nabla I - D)$$

$A = \frac{1}{F - B}$  $D = \alpha \nabla F - (1 - \alpha) \nabla B$

Global Case with Assumption

$$\nabla \alpha \approx \frac{\nabla I}{F - B}$$

$D = \alpha \nabla F - (1 - \alpha) \nabla B = 0$

– i.e. assumes “smooth” foreground and background

Intuitively

– Matte gradient is approximately proportional to image gradient
What does ‘smooth’ mean?

\[ \nabla I = (F - B) \nabla \alpha + \alpha \nabla F + (1 - \alpha) \nabla B \]
\[ D = \alpha \nabla F - (1 - \alpha) \nabla B = 0 \]

Color Image

Grayscale Image

Solve for alpha using smooth assumption, D=0

What should the matte edge be?
What does ‘smooth’ mean?

\[ \nabla I = (F - B)\nabla \alpha + \alpha \nabla F + (1 - \alpha) \nabla B \]

\[ D = \alpha \nabla F - (1 - \alpha) \nabla B = 0 \]

Grayscale \( I \)

\[ \nabla \alpha \approx \frac{\nabla I}{F - B} \]

Gradient \( I \) (in x)

Gradient \( \alpha \) (in x)
What does ‘smooth’ mean?

\[ \nabla I = (F - B)\nabla \alpha + \alpha \nabla F + (1 - \alpha) \nabla B \]

\[ D = \alpha \nabla F - (1 - \alpha) \nabla B = 0 \]

Does this seem like a good matte? Was this “smooth”? 
What does ‘smooth’ mean?

\[ \nabla I = (F - B)\nabla \alpha + \alpha \nabla F + (1 - \alpha) \nabla B \]

\[ D = \alpha \nabla F - (1 - \alpha) \nabla B = 0 \]

What should the matte edge be?
What does ‘smooth’ mean?

\[ \nabla I = (F - B)\nabla \alpha + \alpha \nabla F + (1 - \alpha) \nabla B \]

\[ D = \alpha \nabla F - (1 - \alpha) \nabla B = 0 \]

Does this seem like a good matte? Was this “smooth” assumption good?
Step 1: Global Solution method

This simplified equation is solved in **grayscale** channel

- 2 known variables:
  - gradients of \( I \)
- 3 unknown variables:
  - \((F-B)\), treated as a single variable
  - gradients of \( \alpha \)

\[
\nabla \alpha \approx \frac{\nabla I}{F - B}
\]

+ Additional knowledge

- Boundary conditions from Trimap
Step 1: Global Solution method

Minimize error in

$$\nabla \alpha \approx \frac{\nabla I}{F - B}$$

In the sense of least squares

$$\alpha^* = \arg \min_{\alpha} \int \int_{p \in \Omega} ||\nabla \alpha_p - \frac{1}{F_p - B_p} \nabla I_p||^2 dp$$

This has an associated Poisson equation with Dirichlet boundary conditions (due to Trimap), which we know how to solve

$$\Delta \alpha = \text{div}\left(\frac{\nabla I}{F - B}\right) \quad \text{s.t.} \quad \alpha \big|_{\partial \Omega} = \begin{cases} 1 & x \in \Omega_F \\ 0 & x \in \Omega_B \end{cases}$$

This is solved iteratively, approximating F-B, then solving for $\alpha$, then repeat
Global Poisson Matting

Results good for smooth $F$ and $B$ (thus $D \approx 0$)

Source: http://www.cse.cuhk.edu.hk/~leojia/all_project_webpages/Poisson%20matting/poisson_matting.html
Why do we need local operations?

Exact gradient equation
\[ \nabla \alpha = A (\nabla I - D) \]

\[ A = \frac{1}{F - B} \]
\[ D = \alpha \nabla F - (1 - \alpha) \nabla B \]

Global Poisson matting assumed \( D = 0 \)
Step 2: Local Operations

\[ \nabla \alpha = A(\nabla I - D) \]

Do not assume \( D=0 \)

- **Goal:** change \( A, D \) to produce better approximation of \( \alpha \)
  - ‘Fix’ problematic areas in global result

- **Two categories**
  - Channel Selection
  - Local filters (brushes)

- After local \( \alpha \)-gradient changes solve local Poisson equation with Dirichlet conditions
  - Key: local changes in \( \alpha \)-gradient do not produce artifacts in final matte

\[ \Delta \alpha = \text{div}(A(\nabla I - D)) \]

\[ \hat{\alpha}_p |_{\partial \Omega} = \begin{cases} 
1 & p \in \Omega_F \\
0 & p \in \Omega_B \\
\alpha_g & p \in \Omega
\end{cases} \]
Local Operations: Channel Selection

$$\nabla \alpha = A(\nabla I - D)$$

- Create new weighted channel of R, G, B to minimize $|D|$
  - Instead of grayscale channel
  - Reduces error in D

- Process
  - User selects background samples
  - Automatically calculates channel weights to minimize variance of foreground and background colors
Local Operations: High-pass filter

\[ \nabla \alpha = A(\nabla I - D) \]

- Low pass filter on \( D \)
- High-pass on \( \alpha \) due to \( (\nabla I - D) \) term
  - \( D \) is estimated using low-frequencies of gradient

![Gradient images](Matte gradient before) ![Matte after]
Local Operations: Boosting brush
\[ \nabla \alpha = A(\nabla I - D) \]

- Increase or decrease sharpness
- Gaussian filter on \( A \)

\[ A'_p = [1 + \lambda \exp(-\frac{||p - p_0||^2}{2\sigma^2})] \cdot A_p \]
Local Operations: Diffusion filtering

\[ \nabla \alpha = A(\nabla I - D) \]

• Remove JPEG artifacts
Local Operations: Clone Brush

\[ \nabla \alpha = A(\nabla I - D) \]

- Exactly copy gradient

Matte gradient  |  Matte
Erase brush     |  Inverse brush
Full algorithm summary

0: Create Trimap  
   (user defined)
1: Global Poisson matting  
   (automatic)
2: Apply local Poisson operators  
   (user defined)
   (i) Apply channel selection (smoother background or foreground, reduces |D|)
   (ii) Apply highpass filtering to obtain approximation of D
   (iii) Apply boosting brush to change A
   (iv) Apply clone brush to distinguish gradients

Note: the order of these steps is important! Erase brush and inverse brush can be applied any time
Video break
Full algorithm summary

User (Trimap)

Image

Global Poisson matting

Local Operations

Channel selection

Local filtering
boosting highpass
diffusion cloning

Local Poisson matting

Matte

Source: http://www.cse.cuhk.edu.hk/~leojia/all_project_webpages/Poisson%20matting/poisson_matting.html
Comparison to other methods

- Via picture
Comparison to other methods

- Via picture
Improvements over other methods

• Natural image matting including Bayesian Matting
  – Not easy to adjust output of algorithms if mistakes are made in alpha matte

• Difference matting and triangulation matting
  – Need multiple input images
Applications

- Image compositing
Other applications

• De-fogging
Other Applications

• Multi-background
Thank you!

Questions?
Discussion
Limitations

1.) User-defined local operations
   - Relies on the user for input
   - Good enough?
   - Background knowledge of brushes

2.) Specific order of operations
   - Restricts freedom/flexibility

3.) Similar foreground/background elements
   - Hard to distinguish underlying structure of matte
   - Difficult for computers as well as humans

4.) Multiple distinct foreground elements
   - Requires complicated trimap
   - Bayesian could solve relatively simply

*First two points could be argued as positive qualities as well.*
Improvements

- Ideas?
  - Separate “hair” from “fur”
    - These two come up the most often
    - Have very different properties
Future Work

- Extends naturally to...
  - Video matting!
    - “Real-Time Video Matting Using Multichannel Poisson Equations” - 2010
    - Automatically generated trimaps

- Combination of Poisson and Bayesian Matting
  - Combine user-defined local operations with the pixel by pixel estimation in Bayes'