Poisson Matting

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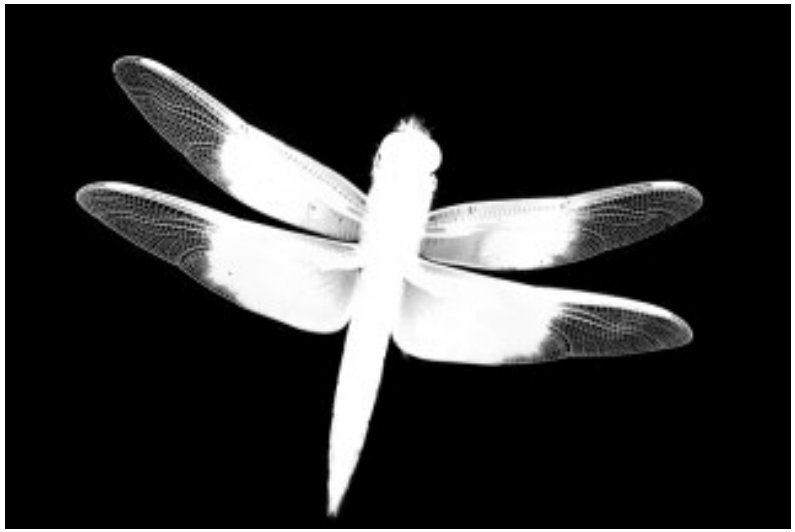
Recall: Matting

- Matting equation over image

$$I = \alpha F + (1 - \alpha) B$$

  - *F* – foreground
  - *B* – background
  - *α* – alpha matte

Image *I*



Resulting alpha matte

Source: http://www.cse.cuhk.edu.hk/~leojia/all_project_webpages/Poisson%20matting/poisson_matting.html

Recall: Matting

- Matting equation over image

$$I = \alpha F + (1 - \alpha) B$$

- Under-constrained equation:
  - 7 unknown variables
    - α
    - R,G,B channels of *F*
    - R,G,B channels of *B*
  - 3 known variables
    - R,G,B channels of *I*
- Additional information
  - User-defined Trimap

Image *I*

Trimap

Poisson Matting

Another approach to solving the matting problem

Approach:

- Use gradient of image to calculate gradient of matte
- Reconstruct matte from gradient of matte

Key addition:

- Enables user interaction to locally improve results

Poisson Matting

- Matting equation

$$I = \alpha F + (1-\alpha)B$$

- Gradient of matting equation

$$\nabla I = (F-B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

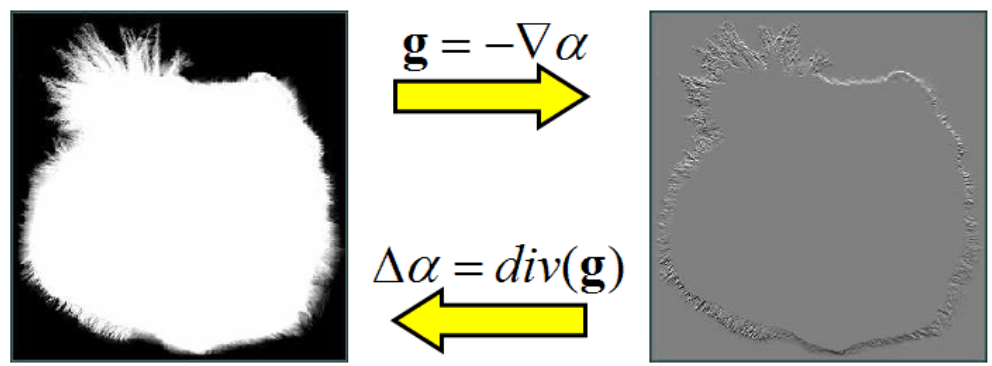
- Gradient rewritten

$$\nabla\alpha = \frac{1}{F-B}(\nabla I - \alpha\nabla F - (1-\alpha)\nabla B)$$

$$A = \frac{1}{F-B}$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B$$

- Once gradient of α is obtained, we can create matte



Poisson Matting

- Matting equation

$$I = \alpha F + (1-\alpha)B$$

- Gradient of matting equation

$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

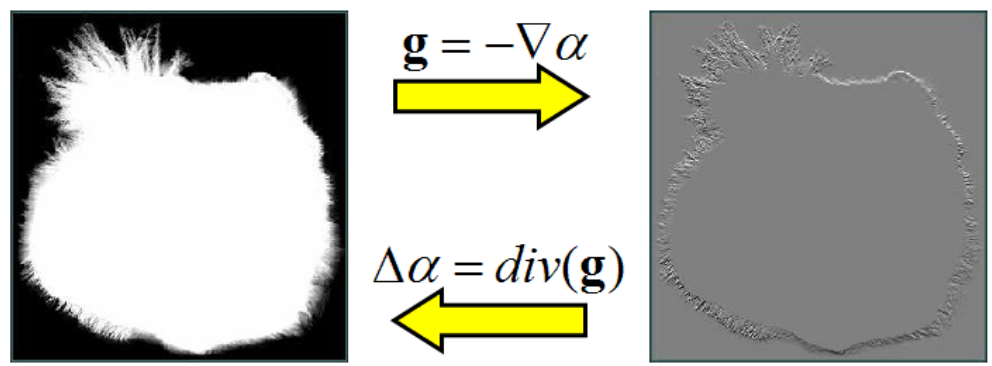
- Gradient rewritten

$$\nabla\alpha = A(\nabla I - D)$$

$$A = \frac{1}{F - B}$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B$$

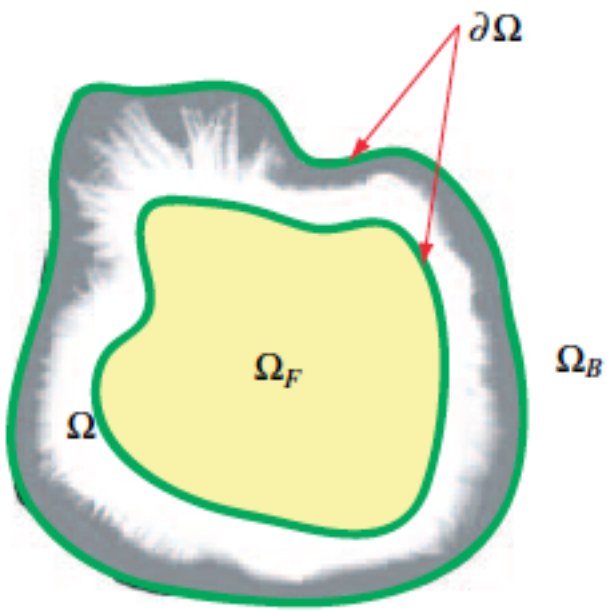
- Once gradient of α is obtained, we can create matte



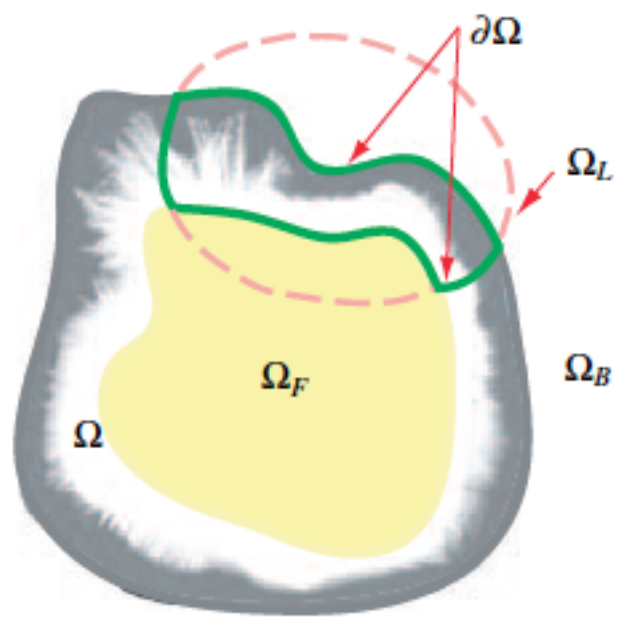
Poisson Matting: Two steps

$$\nabla\alpha = A(\nabla I - D)$$

- Step 1: Global (required)
  - Solve global Poisson matting problem, assumes $D = 0$
- Step 2: Local (optional)
  - Adjust local regions with local Poisson matting operations



(1) Global Poisson Matting



(2) Local Poisson Matting

Step 1: Global Poisson Matting

Global solution assumes ***D*** = 0 in gradient equation

General Case

$$\nabla\alpha = A(\nabla I - D)$$

$$A = \frac{1}{F - B} \qquad D = \alpha\nabla F - (1-\alpha)\nabla B$$

Global Case with Assumption

$$\nabla\alpha \approx \frac{\nabla I}{F - B}$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B = 0$$

- i.e. assumes "smooth" foreground and background

Intuitively

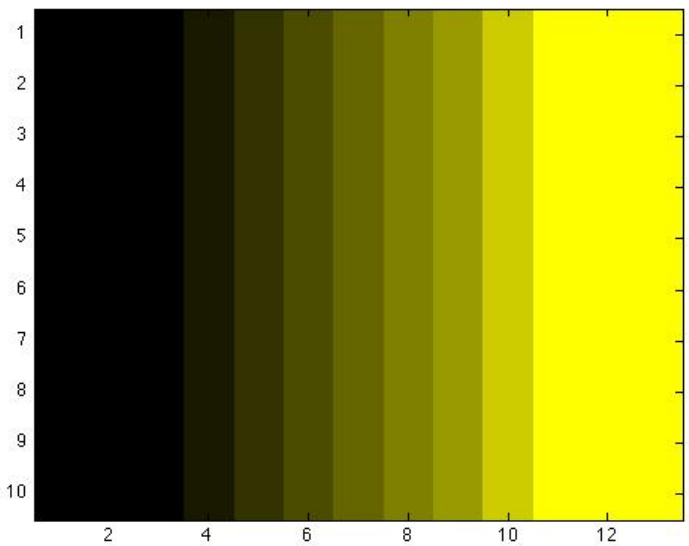
- Matte gradient is approximately proportional to image gradient

What does ‘smooth’ mean?

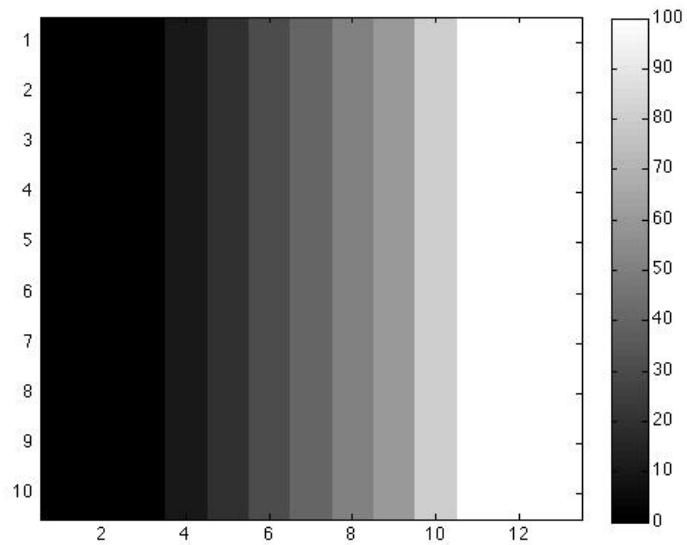
$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B = 0$$

Color Image



Grayscale I



Solve for alpha using smooth assumption, D=0

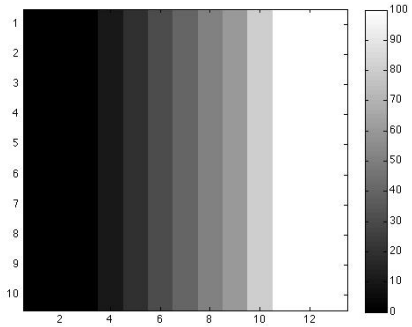
What should the matte edge be?

What does ‘smooth’ mean?

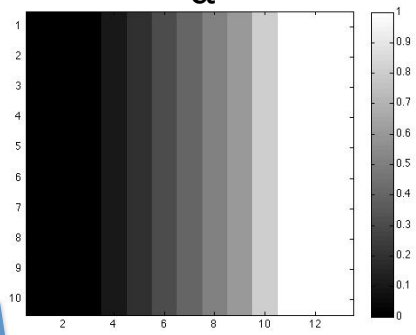
$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B = 0$$

Grayscale I

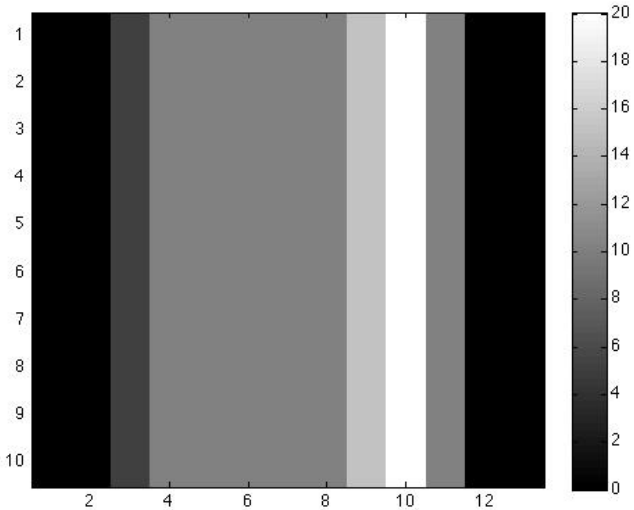


α

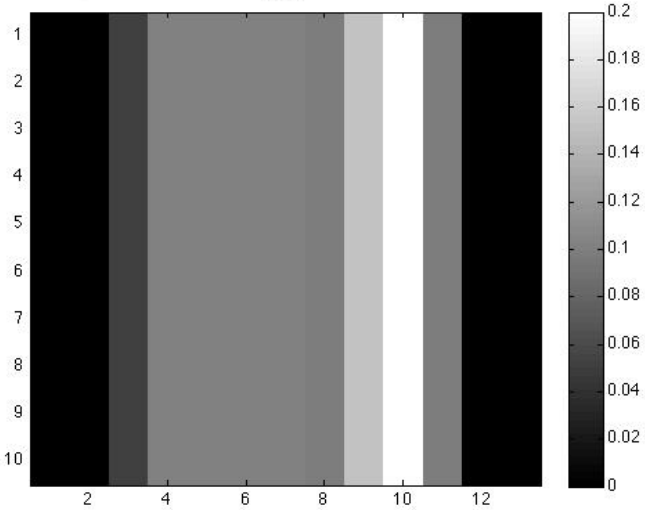


$$\nabla\alpha \approx \frac{\nabla I}{F - B}$$

Gradient I (in x)



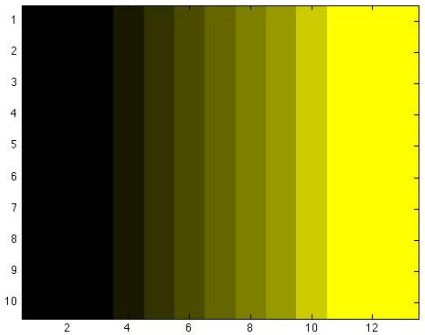
Gradient α (in x)

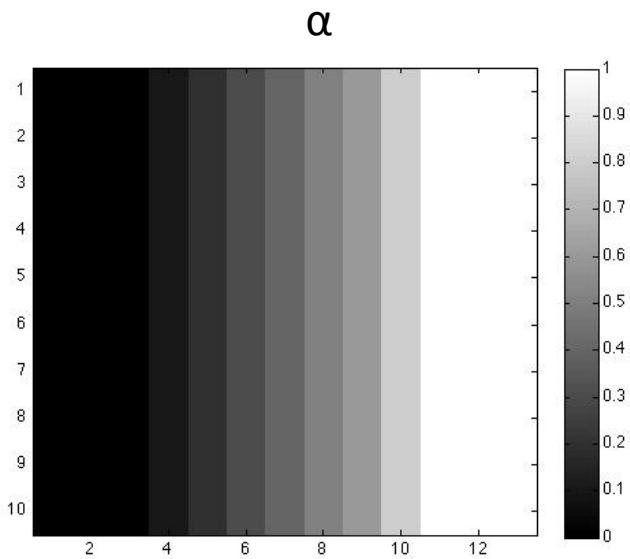


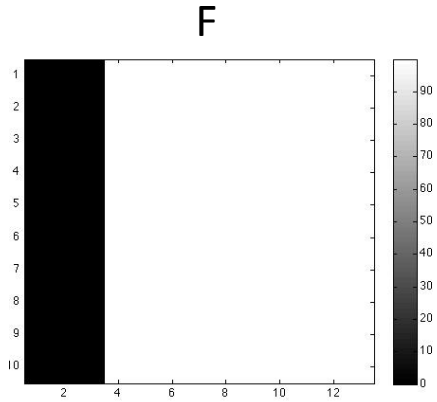
What does ‘smooth’ mean?

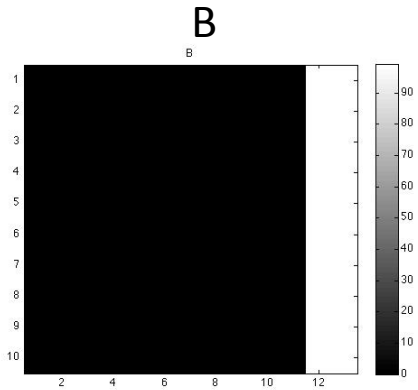
$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B = 0$$









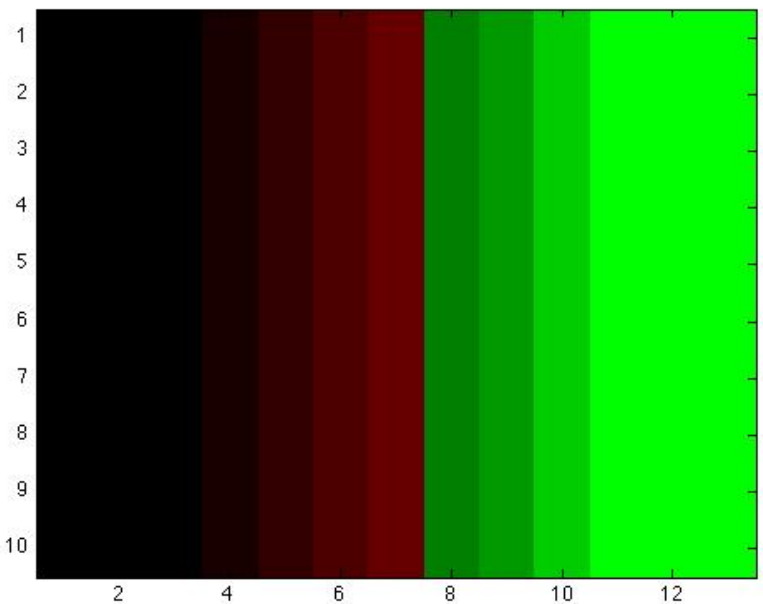
Does this seem like a good matte? Was this “smooth”?

What does ‘smooth’ mean?

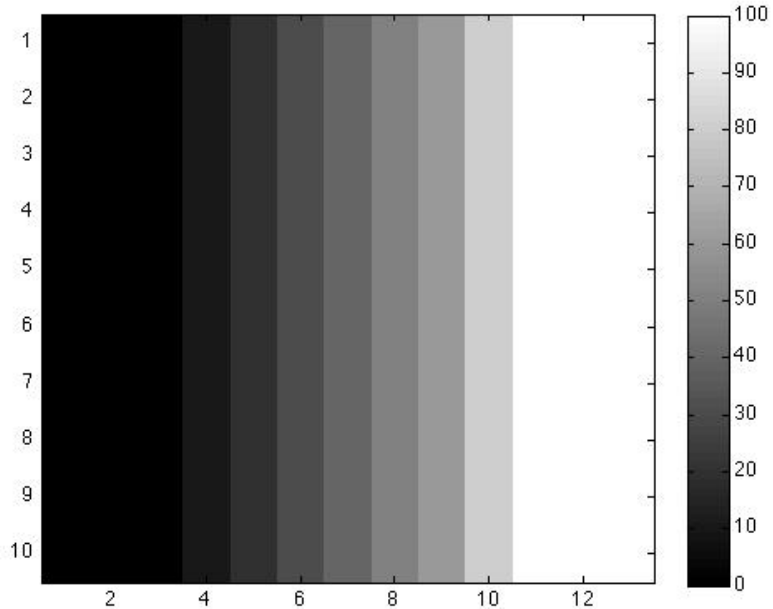
$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B = 0$$

Color Image



Grayscale I



What should the matte edge be?

What does ‘smooth’ mean?

$$\nabla I = (F - B)\nabla\alpha + \alpha\nabla F + (1-\alpha)\nabla B$$

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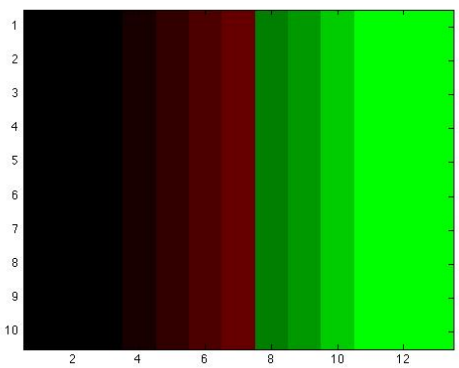
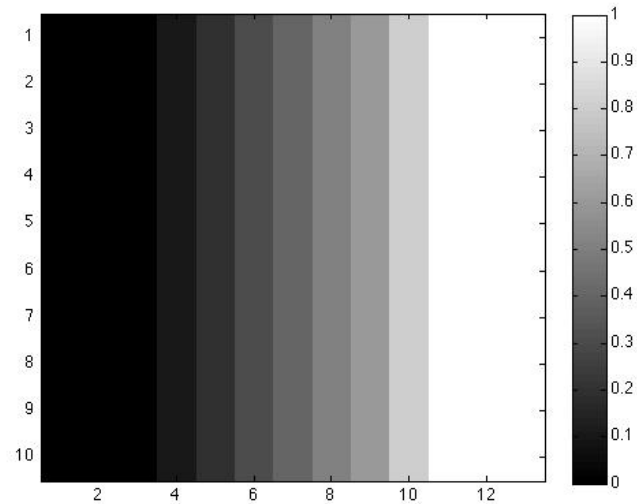
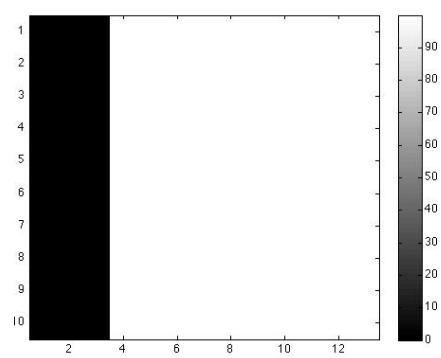
Color Image



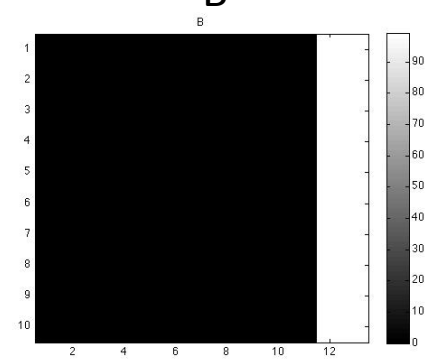
α



F



B



Does this seem like a good matte? Was this “smooth” assumption good?

Step 1: Global Solution method

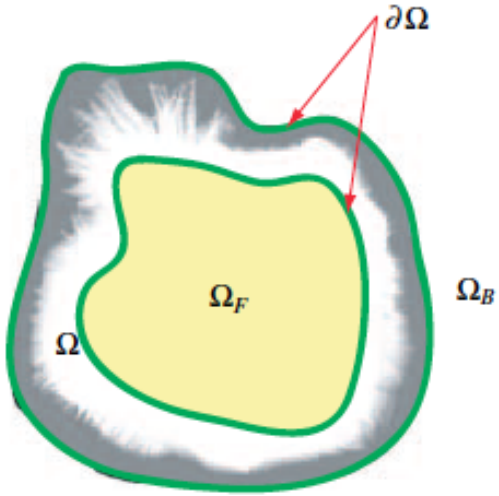
This simplified equation is solved in **grayscale** channel

- 2 known variables:
  - gradients of *I*
- 3 unknown variables:
  - (F-B), treated as a single variable
  - gradients of α

$$\nabla\alpha \approx \frac{\nabla I}{F - B}$$

+ Additional knowledge

- Boundary conditions from Trimap



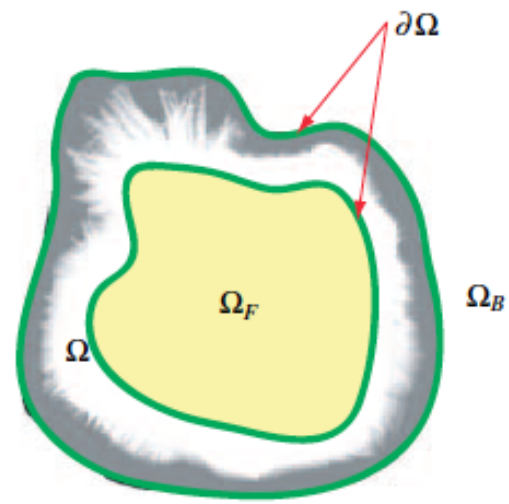
Step 1: Global Solution method

Minimize error in

$$\nabla \alpha \approx \frac{\nabla I}{F - B}$$

In the sense of least squares

$$\alpha^* = \arg\min_{\alpha} \int\int_{p \in \Omega} ||\nabla \alpha_p - \frac{1}{F_p - B_p} \nabla I_p||^2 dp$$



This has an associated Poisson equation with Dirichlet boundary conditions (due to Trimap), which we know how to solve

$$\Delta \alpha = div(\frac{\nabla I}{F - B}) \quad \text{s.t. } \alpha \,|_{\partial\Omega} = \begin{cases} 1 & \mathbf{x} \in \Omega_F \\ 0 & \mathbf{x} \in \Omega_B \end{cases}$$

This is solved iteratively, approximating F-B, then solving for α, then repeat

Global Poisson Matting

Results good for smooth F and B (thus **D**≈0)

Source: http://www.cse.cuhk.edu.hk/~leojia/all_project_webpages/Poisson%20matting/poisson_matting.html

Why do we need local operations?

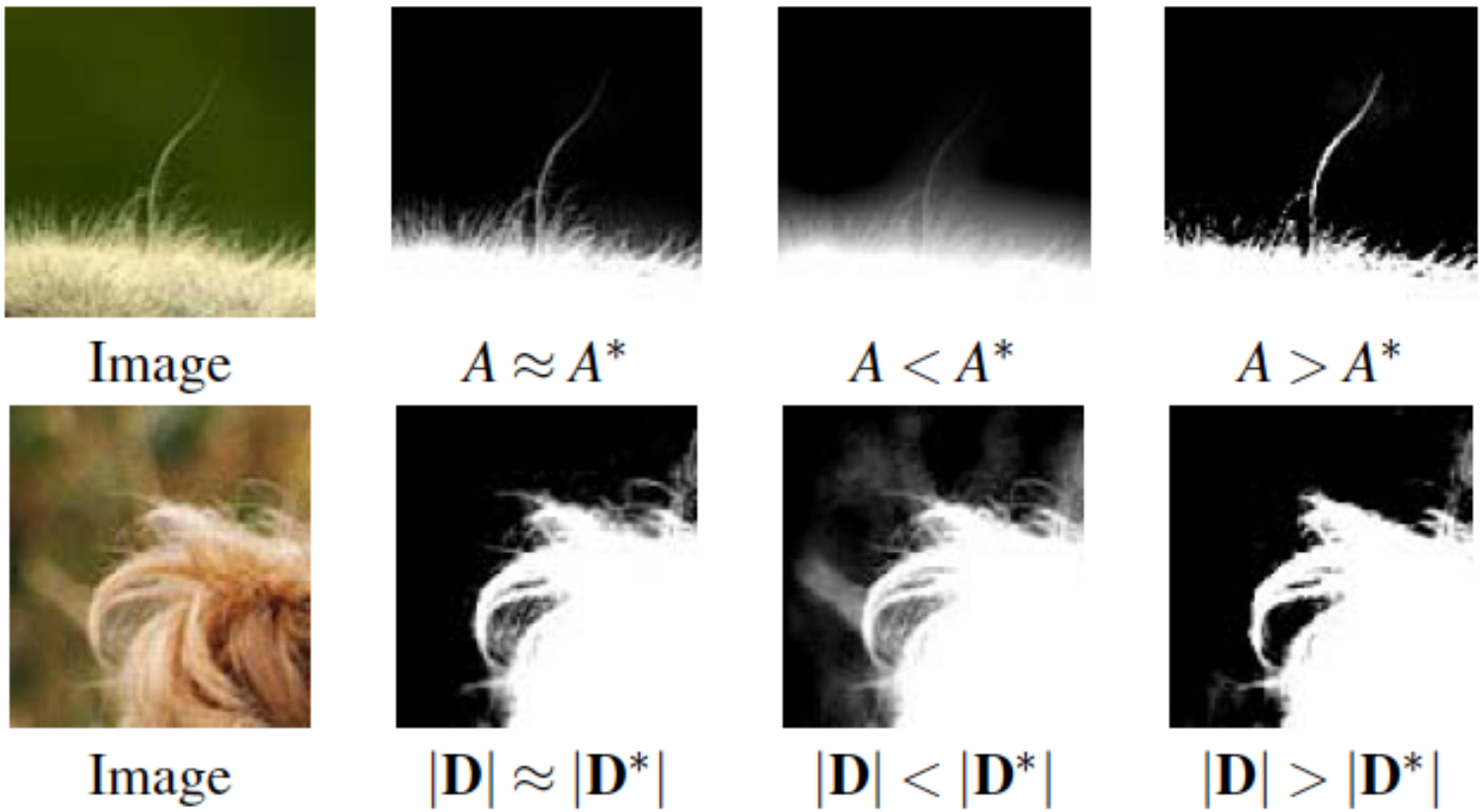
Exact gradient equation

$$\nabla\alpha = A(\nabla I - D)$$

$$A = \frac{1}{F - B}$$

$$D = \alpha\nabla F - (1-\alpha)\nabla B$$

Global Poisson matting assumed D=0

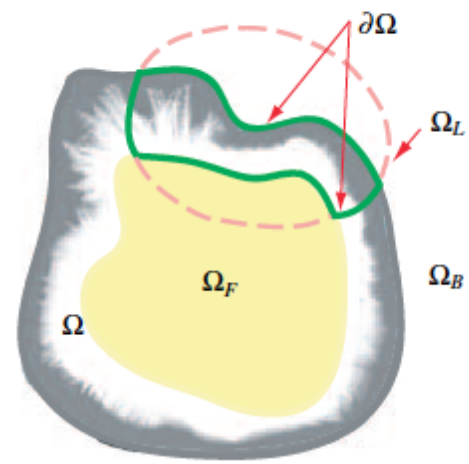


Step 2: Local Operations

$$\nabla\alpha = A(\nabla I - D)$$

Do not assume *D*=0

- **Goal**: change *A*, ***D*** to produce better approximation of **α**
  - 'Fix' problematic areas in global result
- Two categories
  - Channel Selection
  - Local filters (brushes)



- After local α-gradient changes solve local Poisson equation with Dirichlet conditions
  - Key: local changes in α-gradient do not produce artifacts in final matte

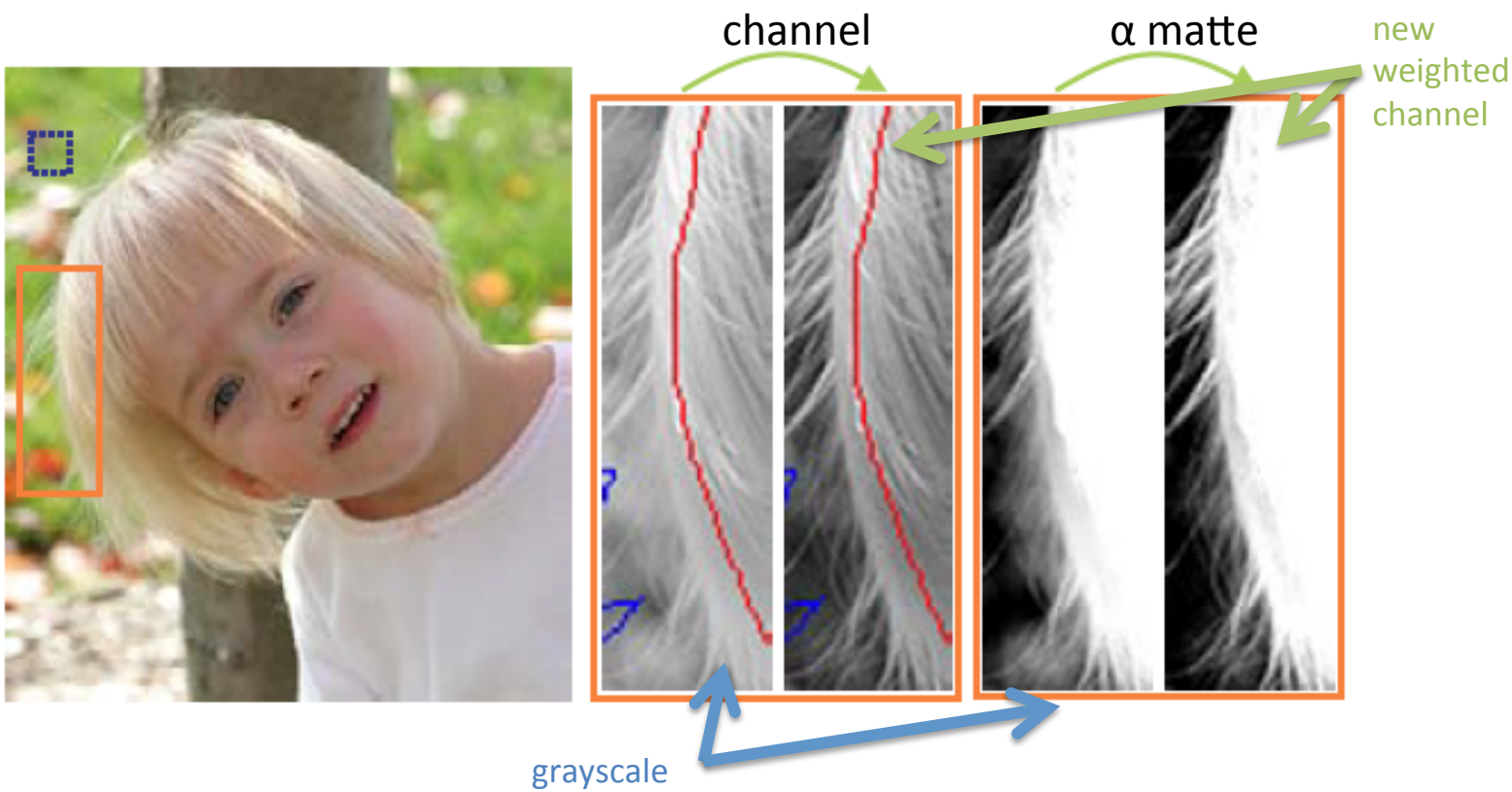
$$\Delta\alpha = div(A(\nabla I - D))$$

$$\widehat{\alpha}_p|_{\partial\Omega} = \begin{cases} 1 & p \in \Omega_F \\ 0 & p \in \Omega_B \\ \alpha_g & p \in \Omega \end{cases}$$

Local Operations: Channel Selection

$$\nabla\alpha = A(\nabla I - D)$$

- Create new weighted channel of R, G, B to minimize $|D|$
  - Instead of grayscale channel
  - Reduces error in D
- Process
  - User selects background samples
  - Automatically calculates channel weights to minimize variance of foreground and background colors



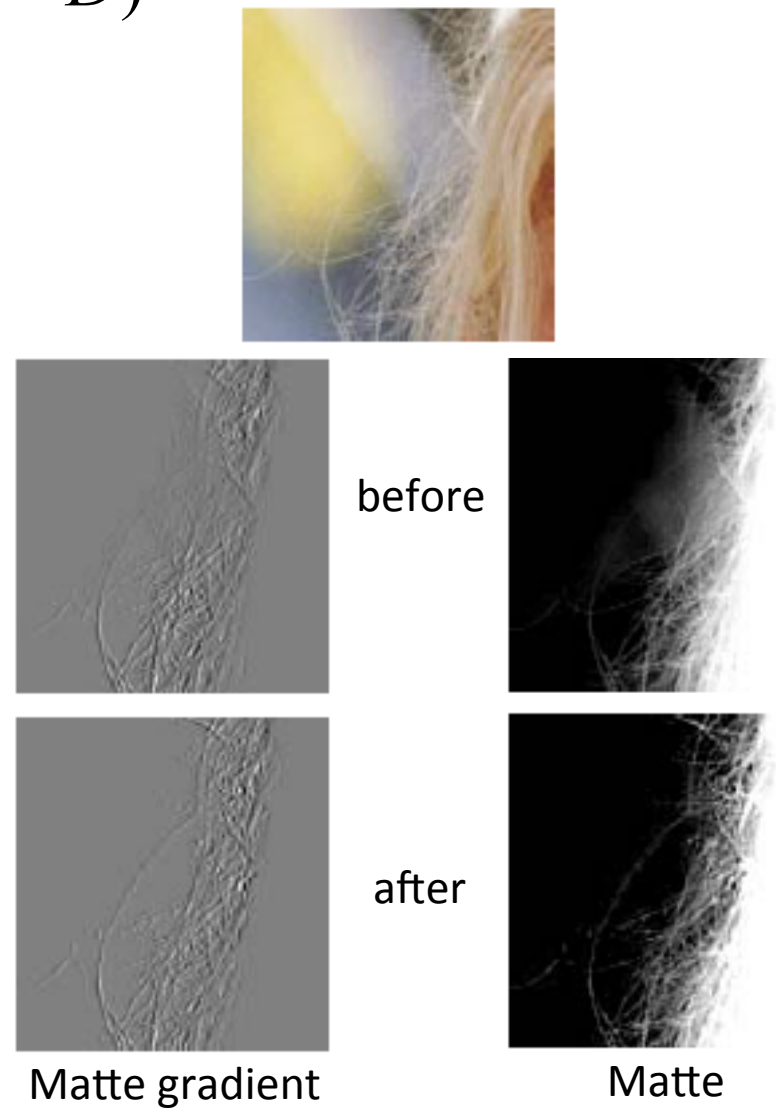
Local Operations: High-pass filter

$$\nabla\alpha = A(\nabla I - D)$$

- Low pass filter on *D*
- High-pass on α due to

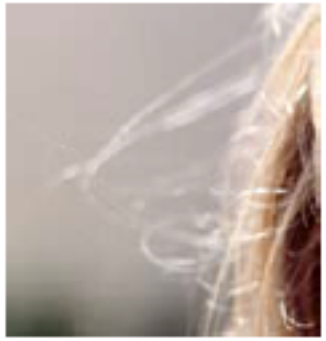
  $(\nabla I - D)$ term

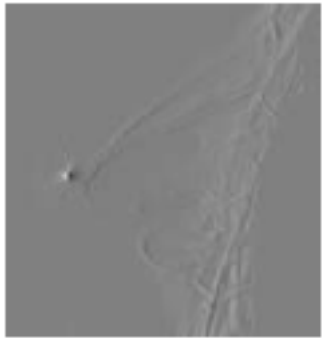
  – *D* is estimated using low-frequencies of gradient



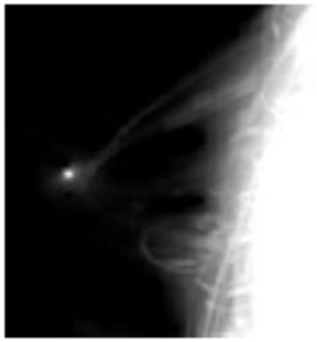
Local Operations: Boosting brush

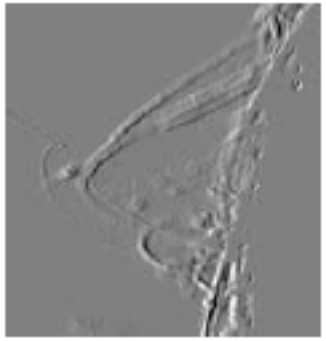
$$\nabla\alpha = A(\nabla I - D)$$



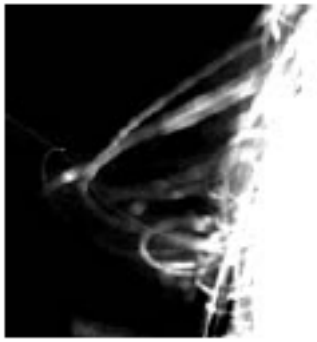


before





after



Matte gradient

Matte

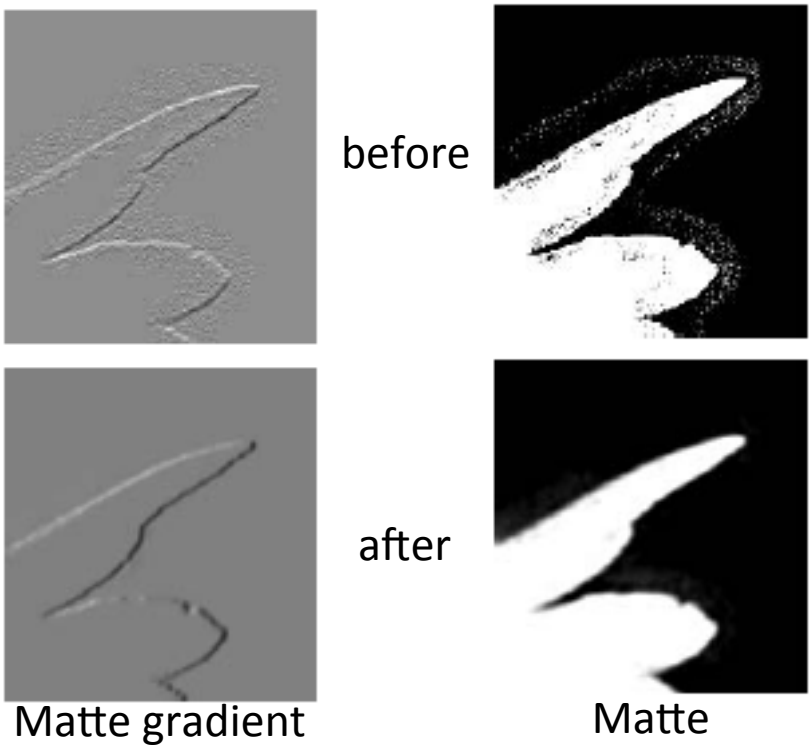
- Increase or decrease sharpness
- Gaussian filter on *A*

$$A'_p = [1 + \lambda \exp(-\frac{||p - p_0||^2}{2\sigma^2})] \cdot A_p$$

Local Operations: Diffusion filtering

$$\nabla\alpha = A(\nabla I - D)$$

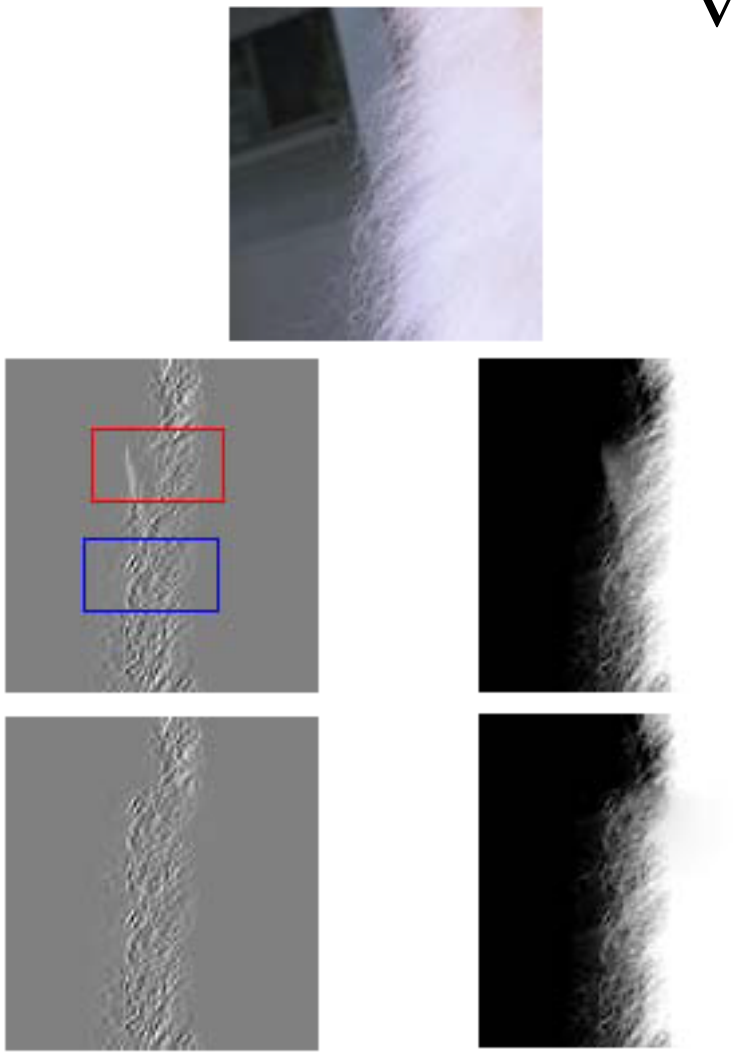
- Remove JPEG artifacts



Local Operations: Clone Brush

$$\nabla\alpha = A(\nabla I - D)$$

- Exactly copy gradient



Matte gradient

Matte

Erase brush

Inverse brush

Full algorithm summary

0: Create Trimap (user defined)

1: Global Poisson matting (automatic)

2: Apply local Poisson operators (user defined)

(i) Apply channel selection (smoother background or foreground, reduces |**D**|)

(ii) Apply highpass filtering to obtain approximation of **D**

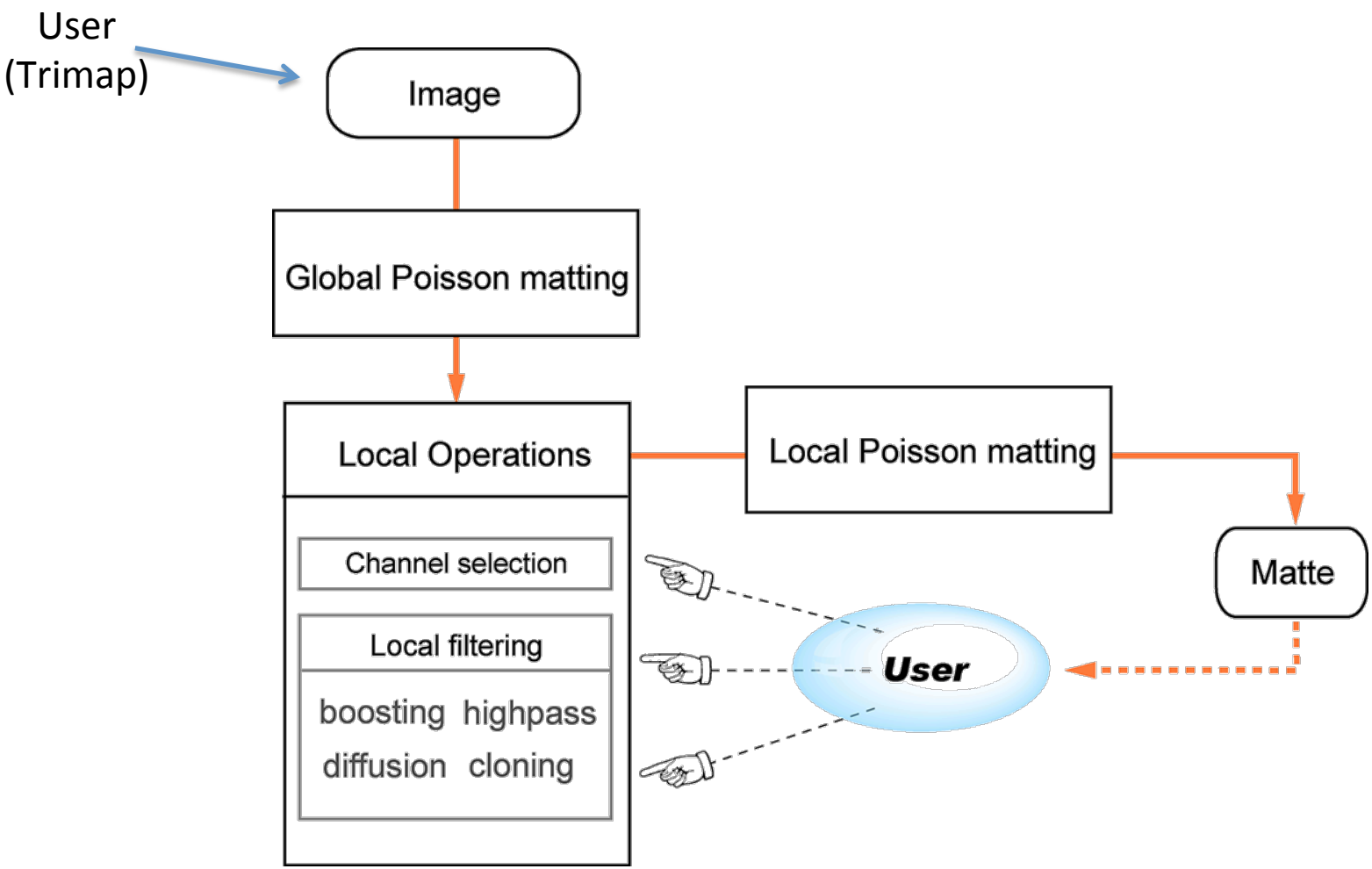
(iii) Apply boosting brush to change A

(iv) Apply clone brush to distinguish gradients

Note: the **order** of these steps is important! Erase brush and inverse brush can be applied any time

Video break

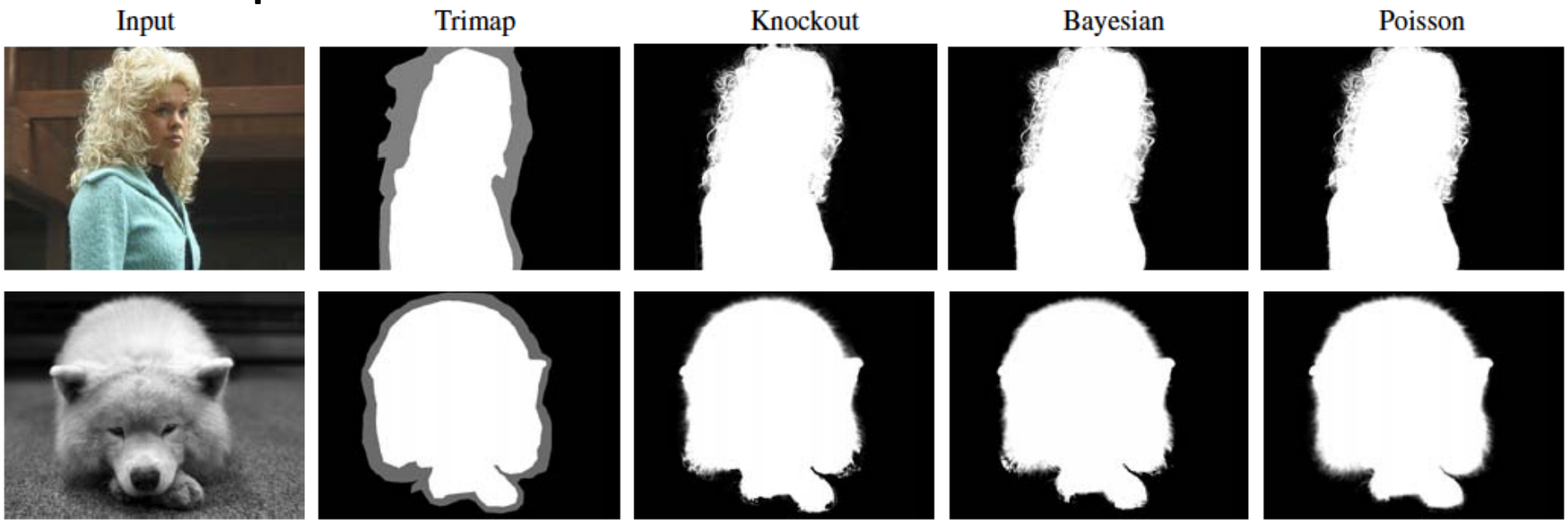
Full algorithm summary



Source: http://www.cse.cuhk.edu.hk/~leojia/all_project_webpages/Poisson%20matting/poisson_matting.html

Comparison to other methods

- Via picture



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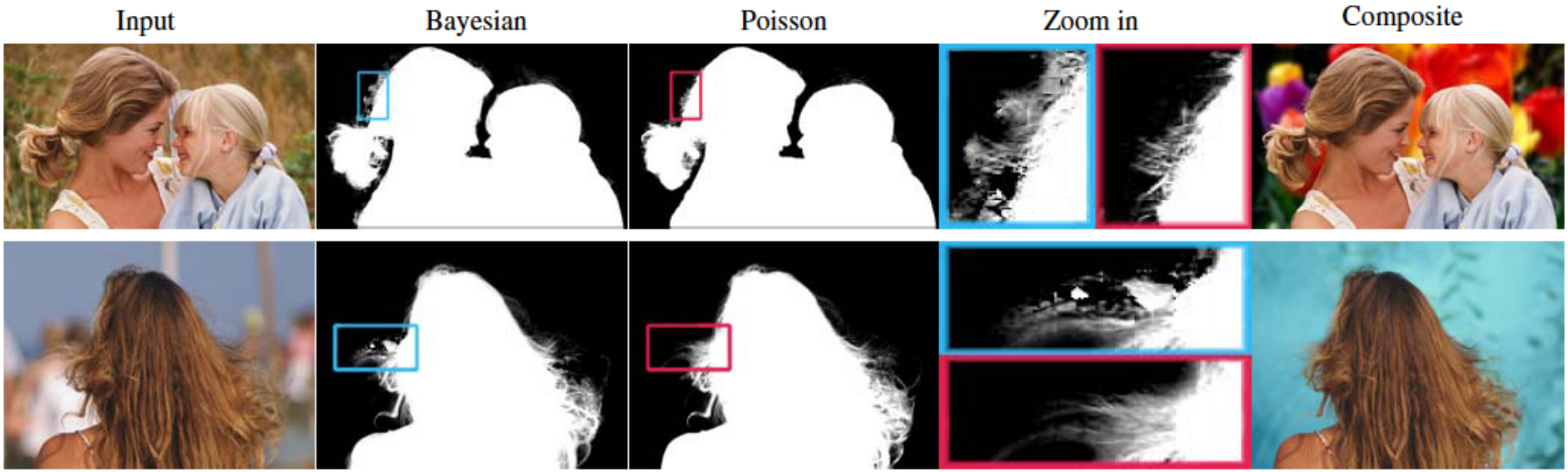

Improvements over other methods

- Natural image matting including Bayesian Matting
  - Not easy to adjust output of algorithms if mistakes are made in alpha matte
- Difference matting and triangulation matting
  - Need multiple input images

Applications

- Image compositing

Other applications

- De-fogging

Other Applications

- Multi-background

Thank you!

Questions?

Discussion

Limitations

1.) User-defined local operations

- Relies on the user for input
- Good enough?
- Background knowledge of brushes

2.) Specific order of operations

- Restricts freedom/flexibility

3.) Similar foreground/background elements

- Hard to distinguish underlying structure of matte
- Difficult for computers as well as humans

4.) Multiple distinct foreground elements

- Requires complicated trimap
- Bayesian could solve relatively simply

*First two points could be argued as positive qualities as well.

Improvements

- Ideas?
  - Separate “hair” from “fur”
    - These two come up the most often
    - Have very different properties

Future Work

- Extends naturally to...

  Video matting!

  - “Real-Time Video Matting Using Multichannel Poisson Equations” - 2010
  - Automatically generated trimaps

- Combination of Poisson and Bayesian Matting
  - Combine user-defined local operations with the pixel by pixel estimation in Bayes'