Graphs and Trees

Jeffrey Heer

CS 294-10: Visualization
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Topics

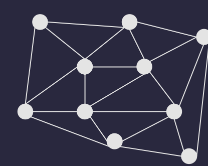
Graph and Tree Visualization
- Tree Layout
- Graph Layout

Goals
- Overview of layout approaches and their strengths and weaknesses
- Insight into implementation techniques

Graphs and Trees

Graphs

- Model relations among data
- *Nodes* and *edges*



Trees

- Graphs with hierarchical structure
  - Connected graph with N-1 edges
- Nodes as *parents* and *children*

Spatial Layout

The primary concern of graph drawing is the spatial layout of nodes and edges

Often (but not always) the goal is to effectively depict the graph structure

- Connectivity, path-following
- Network distance
- Ordering (e.g., hierarchy level)

Applications of Tree / Graph Layout

Tournaments

Organization Charts

Genealogy

Diagramming (e.g., Visio)

Biological Interactions (Genes, Proteins)

Computer Networks

Social Networks

Simulation and Modeling

Integrated Circuit Design

Tree Visualization

Indentation

- Linear list, indentation encodes depth

Node-Link diagrams

- Nodes connected by lines/curves

Enclosure diagrams

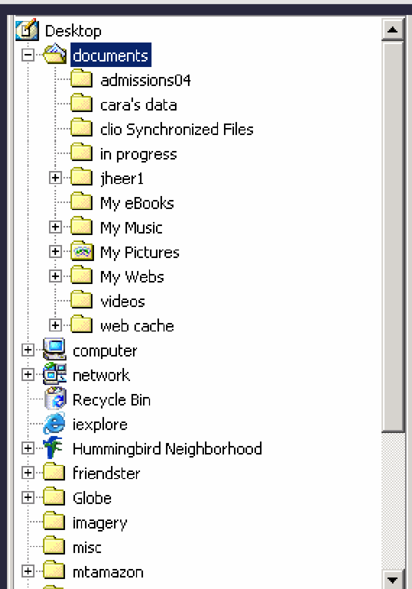
- Represent hierarchy by enclosure

Layering

- Layering and alignment

Tree layout is fast: O(n) or O(n log n), enabling real-time layout for interaction.

Indentation



Places all items along vertically spaced rows

Indentation used to show parent/child relationships

Commonly used for text

Breadth and depth contend for space

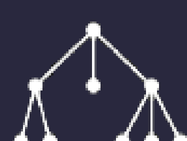
Often requires a great deal of scrolling

Node-Link Diagrams

Nodes are distributed in space, connected by straight or curved lines

Typical approach is to use 2D space to break apart breadth and depth

Often space is used to communicate hierarchical orientation (typically towards authority or generality)

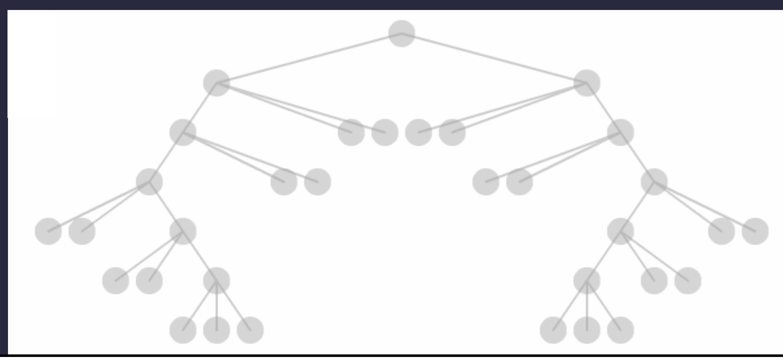


Basic Recursive Approach

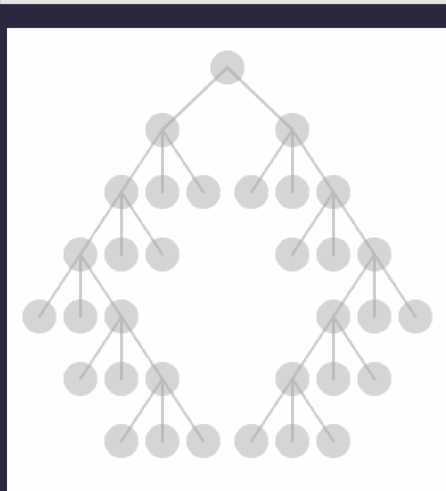
Repeatedly sub-divide space for subtrees

- Breadth of tree along one dimension
- Depth along the other dimension

Problem: exponential growth of breadth



Reingold & Tilford's Tidier Layout



Goal: make smarter use of space, maximize density and symmetry.

Originally for binary trees, extended by Walker to cover general case.

This extension was corrected by Buchheim et al to achieve a linear time algorithm.

Reingold-Tilford Layout

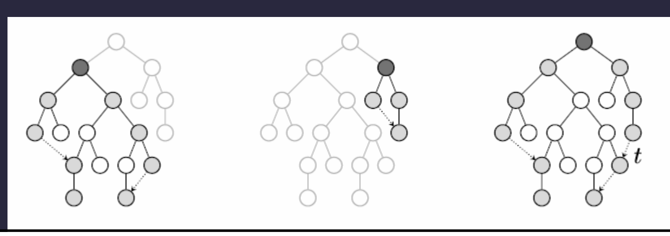
Design concerns

- Clearly encode depth level
- No edge crossings
- Isomorphic subtrees drawn identically
- Ordering and symmetry preserved
- *Compact layout (don't waste space)*

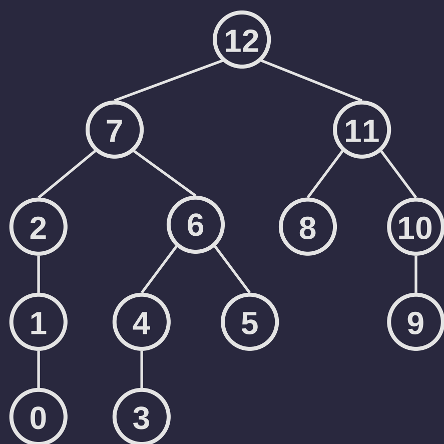
Reingold-Tilford Algorithm

Linear algorithm – starts with bottom-up pass of the tree

- Y-coord by depth, arbitrary starting X-coord
- Merge left and right subtrees
  - Shift right as close as possible to left
    - Computed efficiently by maintaining subtree contours
  - "Shifts" in position saved for each node as visited
  - Parent nodes are centered above their children
- Top-down pass for assignment of final positions
  - Sum of initial layout and aggregated shifts



Reingold-Tilford Algorithm



Reingold-Tilford Algorithm

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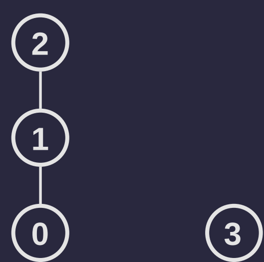

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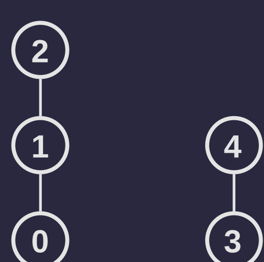

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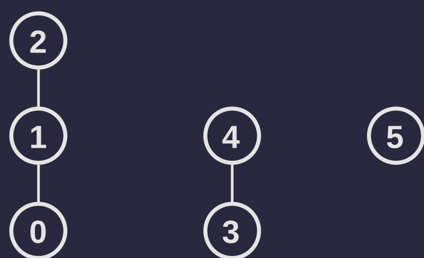

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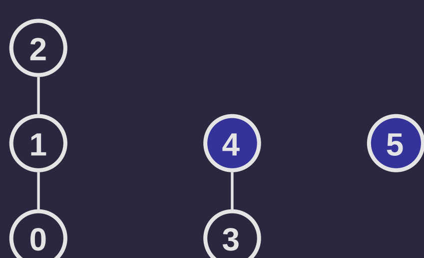

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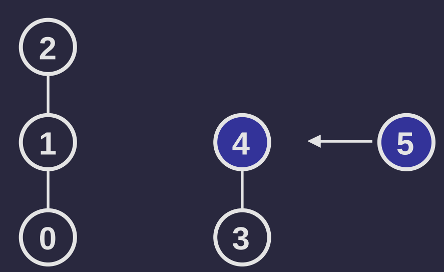

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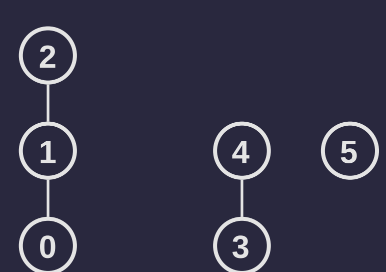

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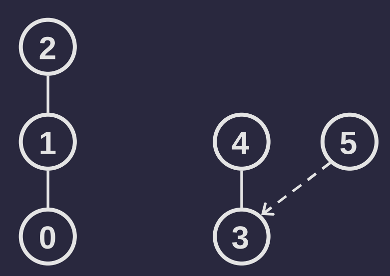

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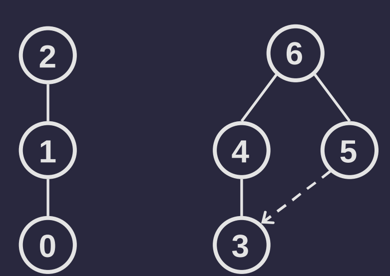

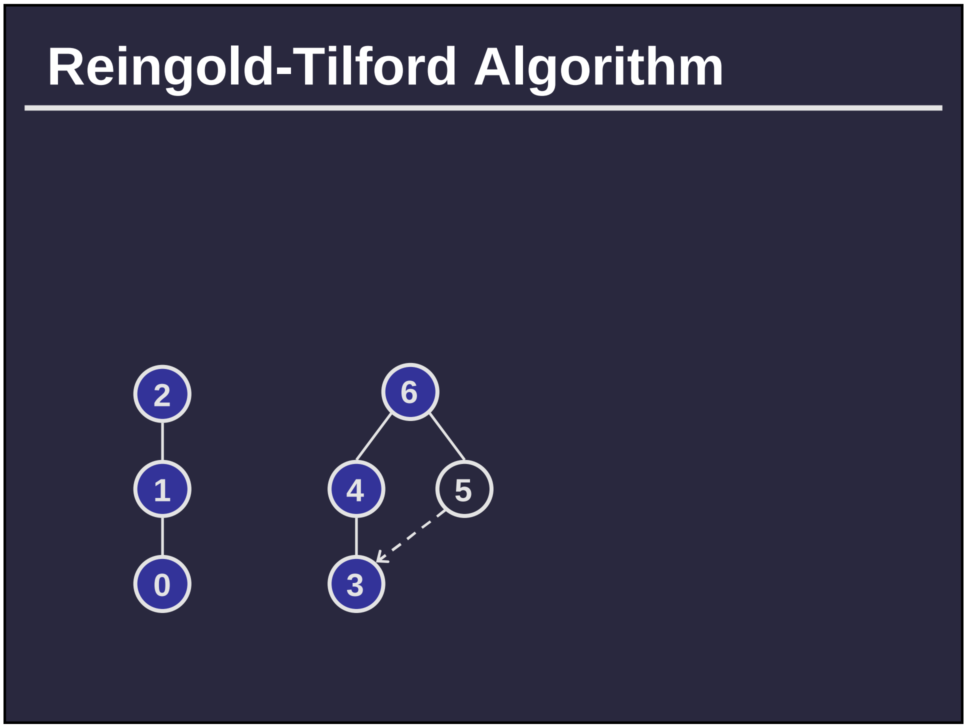
Reingold-Tilford Algorithm
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1
0
6
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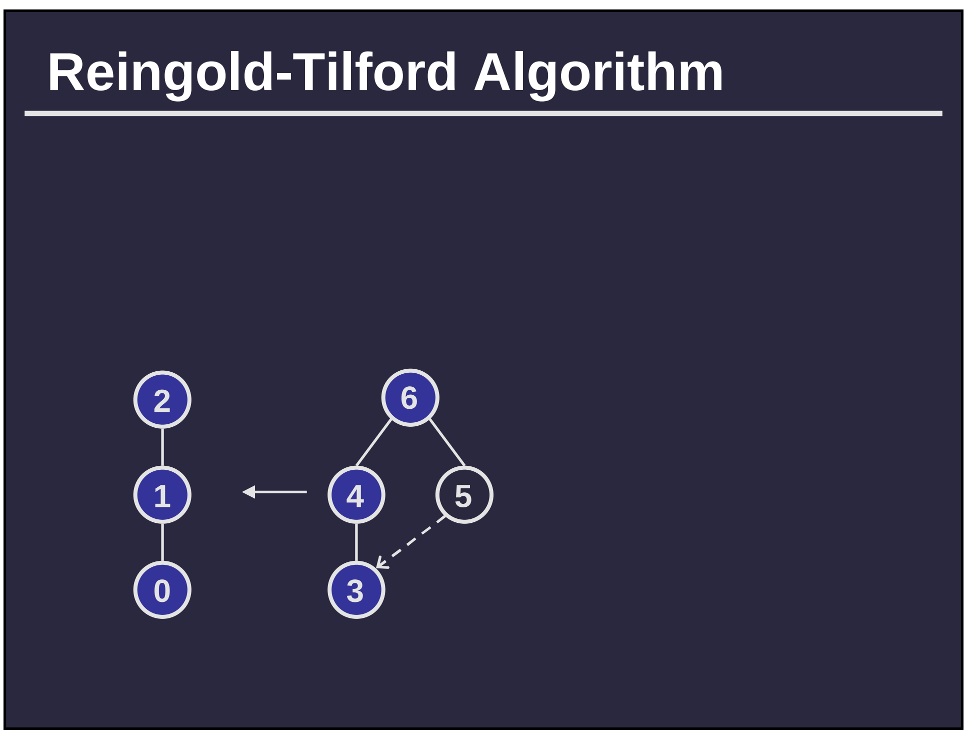
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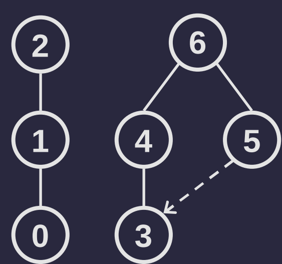

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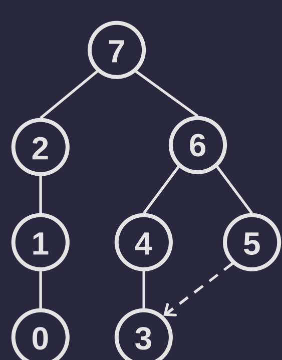

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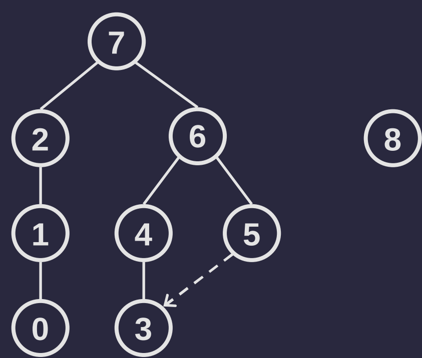

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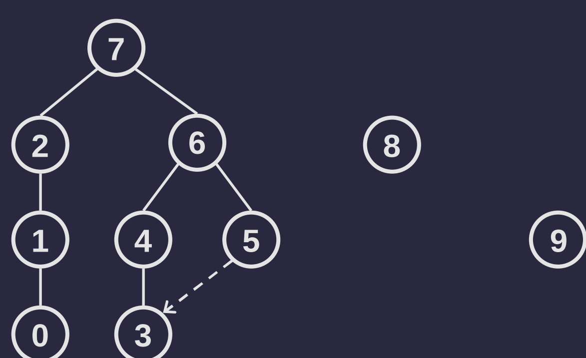

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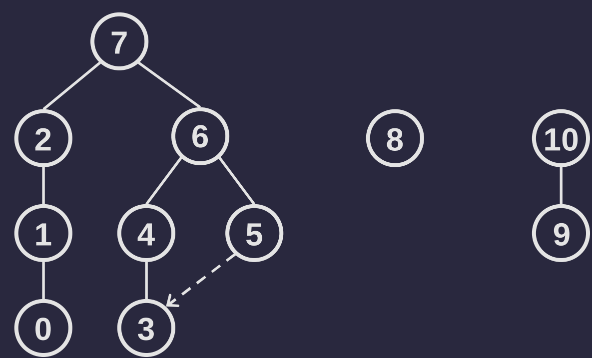

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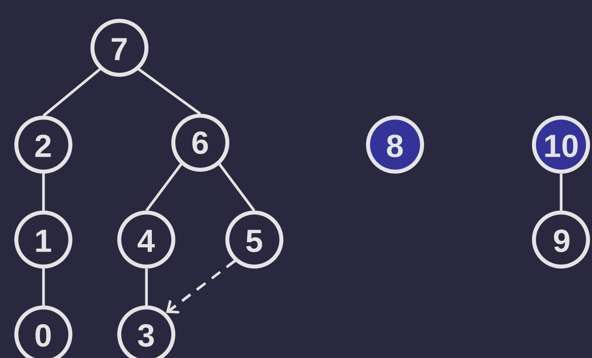

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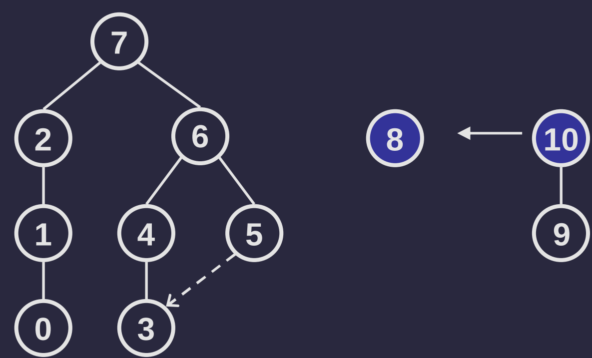

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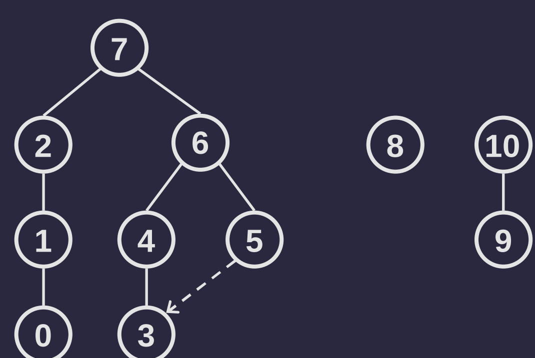

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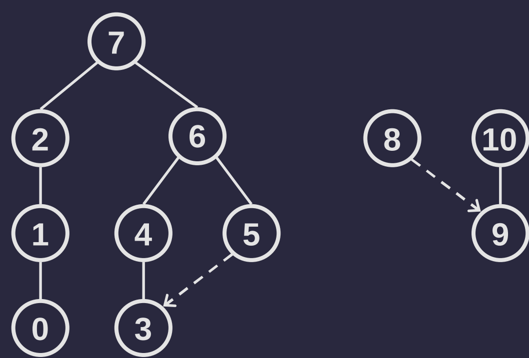

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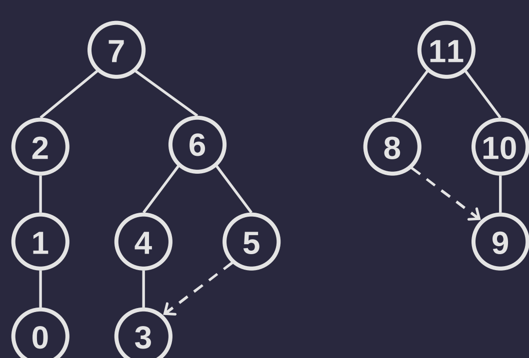

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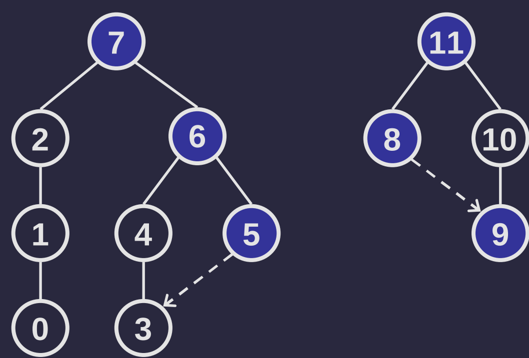

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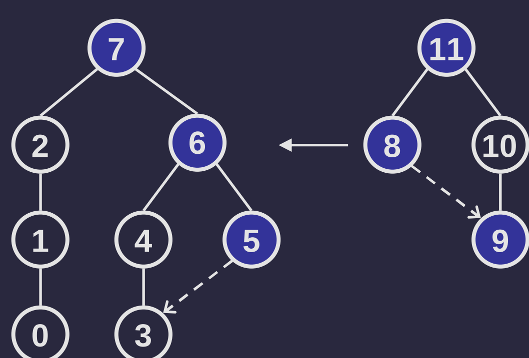

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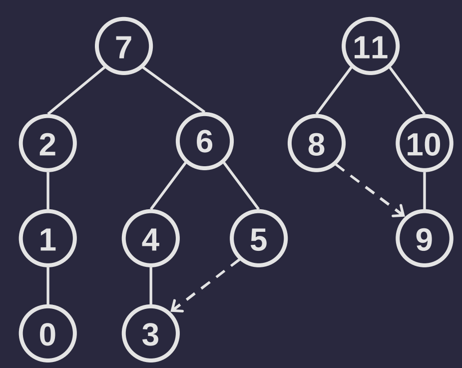

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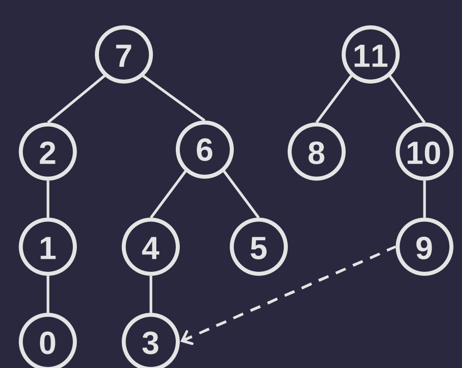

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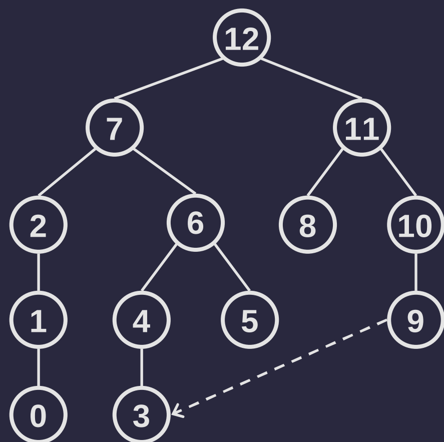
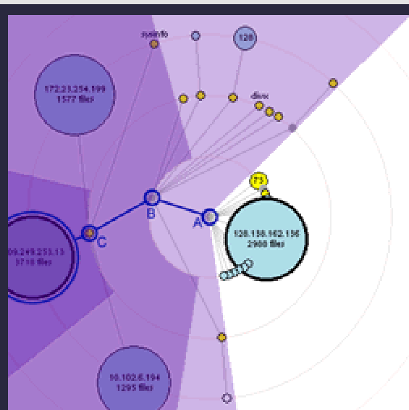


Radial Layout



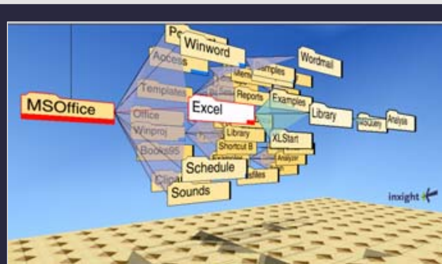
Node-link diagram in polar co-ordinates.

Radius encodes depth, with root in the center.

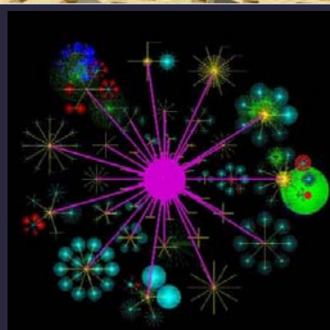
Angular sectors assigned to subtrees (typically uses recursive approach).

Reingold-Tilford approach can also be applied here.

Circular Drawing of Trees



Can be done in three dimensions to form "Cone Trees"

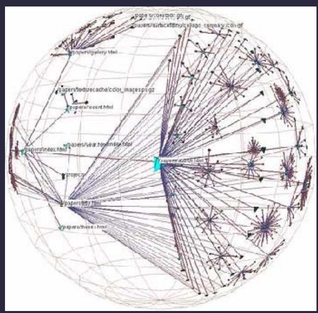


Can also make "Balloon Trees", sometimes described as a 2D version of a Cone Tree. Not just a flattening process, as circles must not overlap.

Problems with Node-Link Diagrams

- Scale
  - Tree breadth often grows exponentially
    - Even with tidier layout, quickly run out of space
- Possible solutions
  - Filtering
  - Focus+Context
  - Scrolling or Panning
  - Zooming
  - Aggregation

Hyperbolic Layout

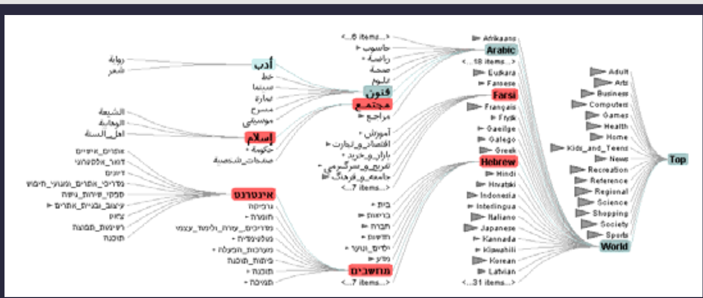


Perform tree layout in hyperbolic geometry, then project the result on to the Euclidean plane.

Why? Like tree breadth, the hyperbolic plane expands exponentially!

Also computable in 3D, projected into a sphere.

Degree-of-Interest Trees

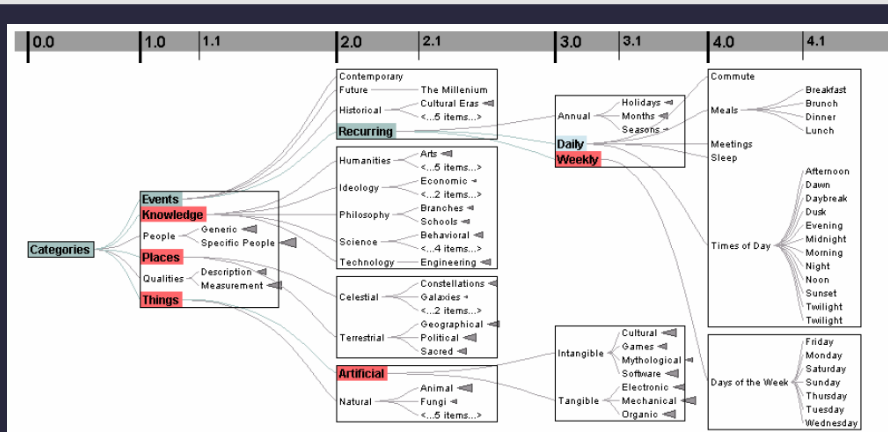


Filter to show only the most "interesting" nodes. Requires "degree-of-interest" (DOI) estimation.

Enforce that breadth does not exceed display. Aggregate siblings to achieve this.

Use glyphs to encode unexpanded subtrees.

Degree-of-Interest Trees



Cull “un-interesting” nodes on a per block basis until all blocks on a level fit within bounds.

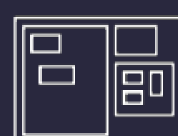
Attempt to center child blocks beneath parents.

Enclosure Diagrams

Signify structure using spatial enclosure

Venn diagrams without intersection

Popularly known as “TreeMaps”



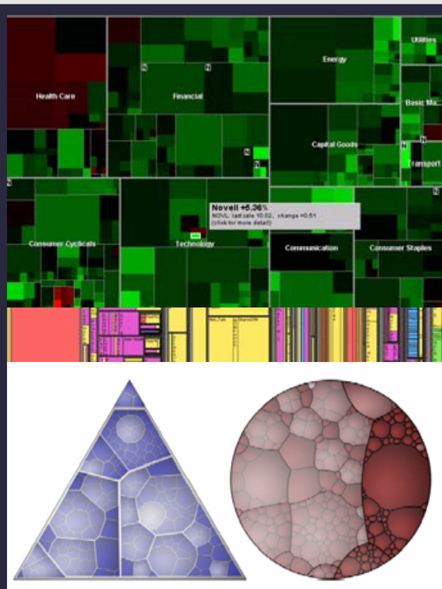
Benefits

- Provides a single view of an entire tree
- Easier to spot large/small nodes

Problems

- Difficult to accurately read depth

TreeMaps



Recursively fill space based on a size metric for nodes. Enclosure signifies hierarchy.

Additional measures can be taken to control aspect ratio of cells.

Often uses rectangles, but other shapes are possible, e.g., iterative Voronoi tesselation.

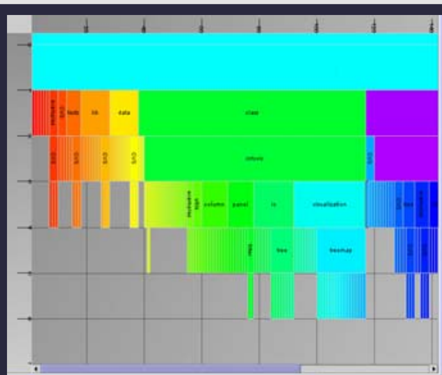
Layered Diagrams

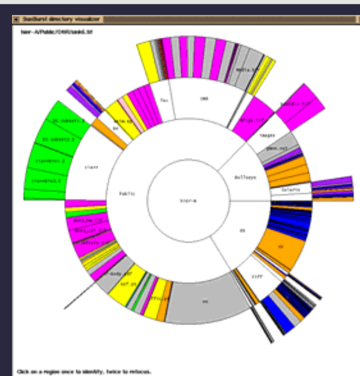
Signify tree structure using

- Layering
- Adjacency
- Alignment

Typically involves recursive sub-division of space – we can apply the same set of approaches as in node-link layout.

Icicle and Sunburst Trees

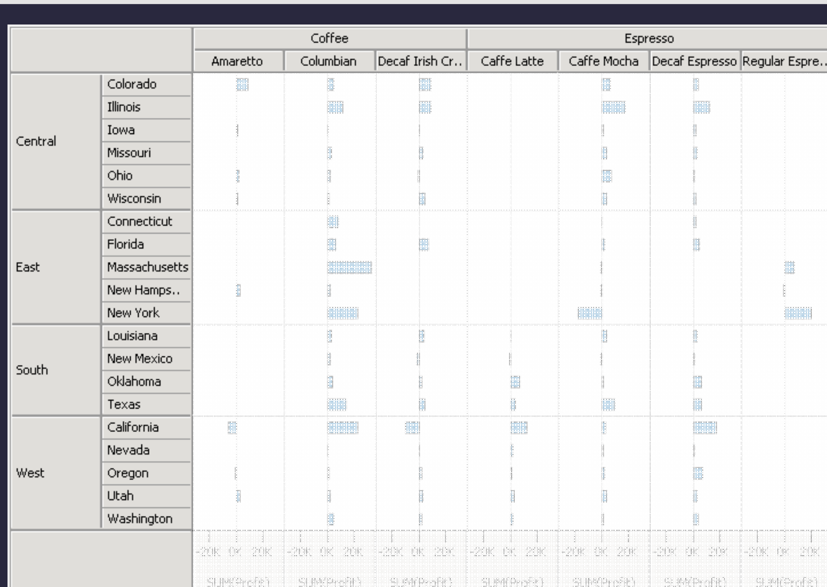




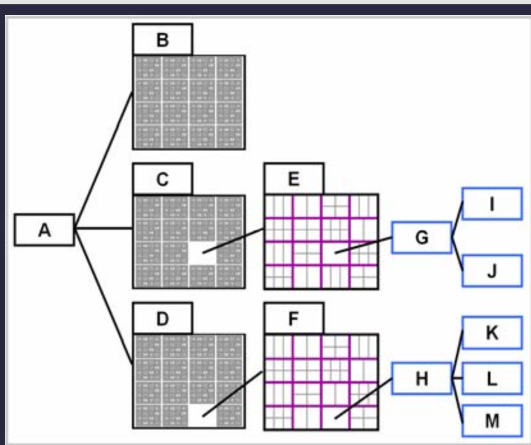
Higher-level nodes get a larger layer area, whether that is horizontal or angular extent.

Child levels are layered, constrained to extent of the parent.

Layered Tree Drawing



Hybrids are also possible…



"Elastic Hierarchies"

Node-link diagram with treemap nodes.

Graph Visualization

Approaches to Graph Drawing

Direct Calculation using Graph Structure

- Tree layout on spanning tree
- Adjacency matrix layout
- Hierarchical layout

Optimization-based Layout

- Constraint satisfaction
- Force-directed layout

Attribute-Driven Layout

- Layout using data attributes, not linkage

Spanning Tree Layout

Many graphs are tree-like or have spanning trees of interest

- Websites, Social Networks

Use tree layout on spanning tree of graph

- Trees created by BFS / DFS
- Min/max spanning trees

Fast tree layouts allow graph layouts to be recalculated at interactive rates

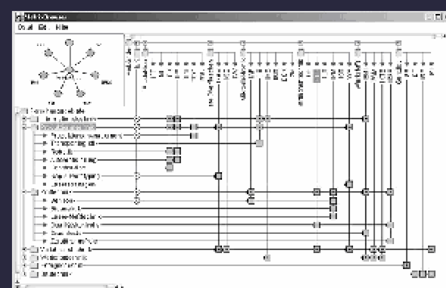
Heuristics may further improve layout

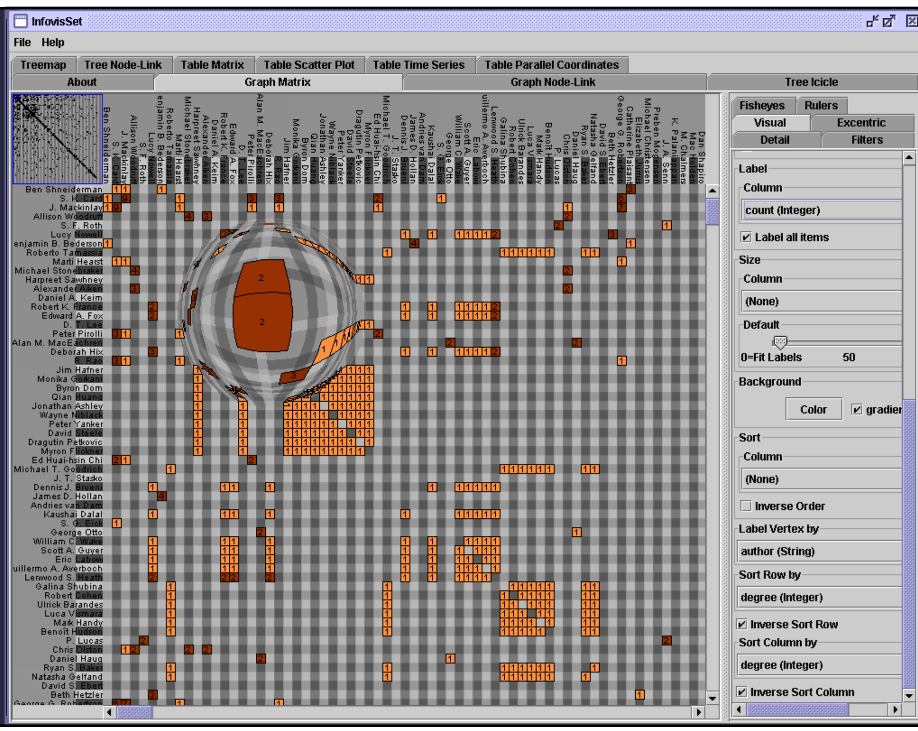
Adjacency Matrices

Node-link diagrams often don't scale well due to edge-crossings, occlusion

One solution: adjacency matrix

- show graph as table
- nodes as rows/columns
- edges as table cells

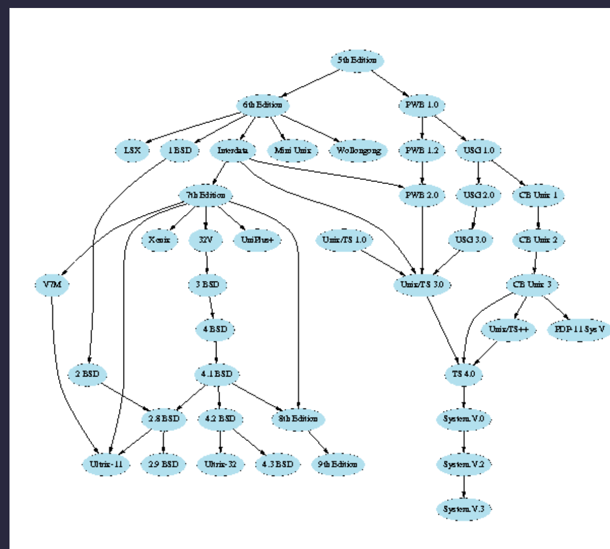




Sugiyama-style graph layout

Evolution of the UNIX operating system

Hierarchical layering based on descent



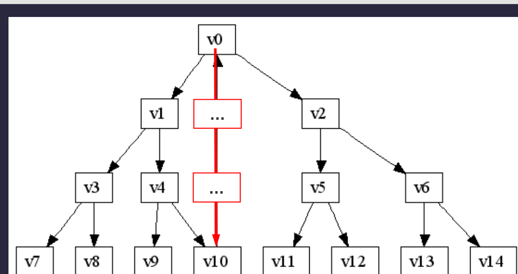
Sugiyama-style graph layout

Layer 1

Layer 2

Layer 3

Layer 4

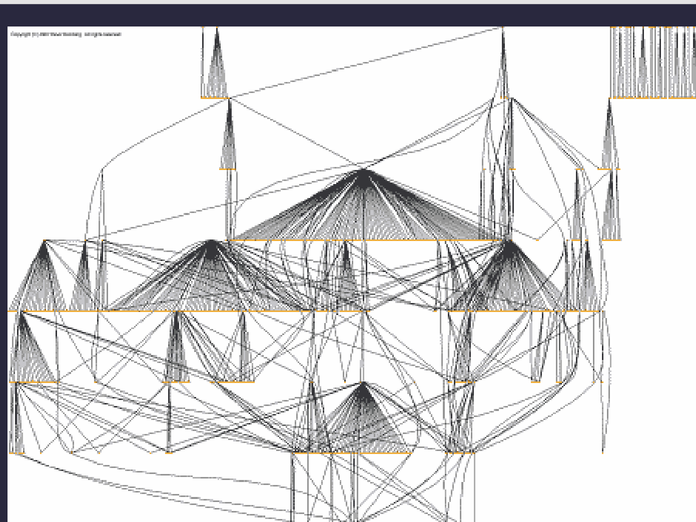


Assign nodes to hierarchy layers
- Reverse edges to remove cycles
- Create dummy nodes to "fill in" missing layers

Arrange nodes within layer
- Often try to minimize edge crossings

Route edges – layout splines if needed

Hierarchical graph layout



Gnutella network

Optimization Techniques

Treat layout as an *optimization problem*

- Define layout using a set of *constraints*: equations the layout should try to obey
- Use optimization algorithms to solve

Common approach for undirected graphs:

- *Force-Directed Layout* most common

We can introduce directional constraints:

- *DiG-CoLa* (Di-Graph Constrained Optimization Layout)
  - Dwyer and Koren, Best Paper at InfoVis 2005

Optimizing "Aesthetic" Constraints

Minimize edge crossings

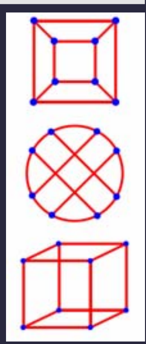
Minimize area

Minimize line bends

Minimize line slopes

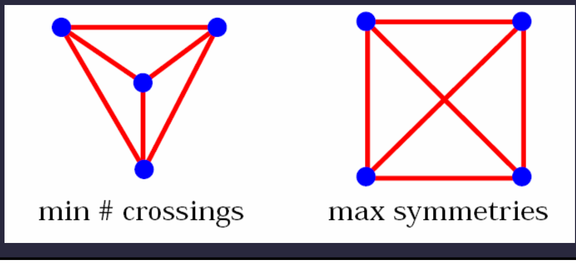
Maximize smallest angle between edges

Maximize symmetry



but, can't do it all.

Optimizing these criteria is often NP-complete, requiring approximations.



Force-Directed Layout

Edges = springs

- F = -k * (x – L)
  - k is the spring tension coefficient (how tight the spring is)
  - L is the rest-length of the spring (the desired edge length)
  - x is the current length of the spring (i.e., distance between nodes)

Nodes = repulsive charged particles

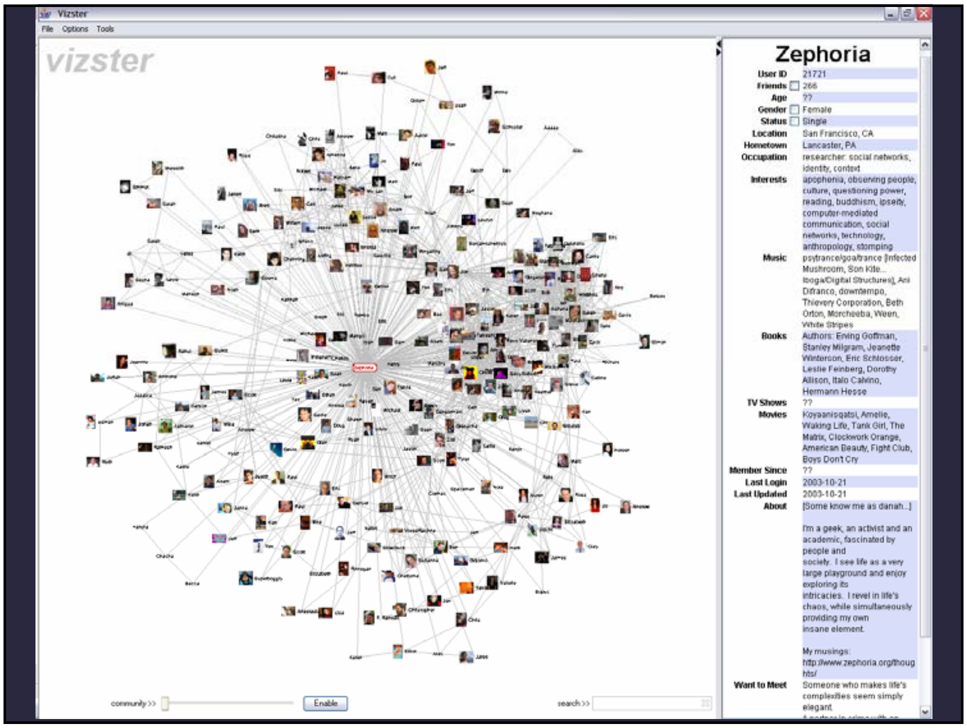
- $F = G*m_1*m_2 / x^2$
  - G is a constant – negative for repulsion!
  - m_1, m_2 are the masses/charges of the nodes
  - x is the distance between nodes

Repeatedly calculate forces, update node positions

- Naïve approach costly: $O(N^2)$ comparisons each iteration
- Speed up to O(N log N) with Barnes-Hut algorithm (quadtree)

We can animate the optimization process

- Requires numerical integration techniques to ensure that updates for each frame are smooth and stable.



Constrained Optimization Layout

Minimize "stress" function

$$\text{stress}(X) = \Sigma_{i<j}\, w_{ij}\, (\, \|X_i - X_j\| - d_{ij}\,)^2$$

- X: node positions, d: optimal edge length,
- w: normalization constants
- Can be subject to global or localized optimization
  - Local: Gradient descent (Kamada-Kawai)
  - Global: Majorization (Gansner, Koren, North)

- Says: Try to place nodes d_{ij} apart

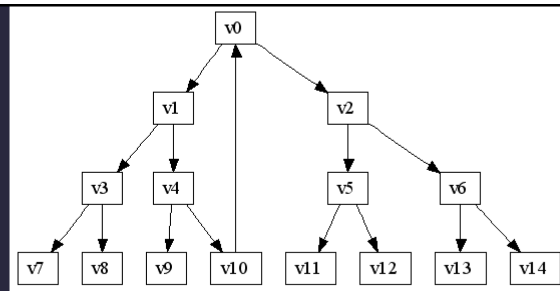
Add hierarchy ordering constraints

$$E_H(y) = \Sigma_{(i,j)\in E}\, (\, y_i - y_j - \delta_{ij}\,)^2$$

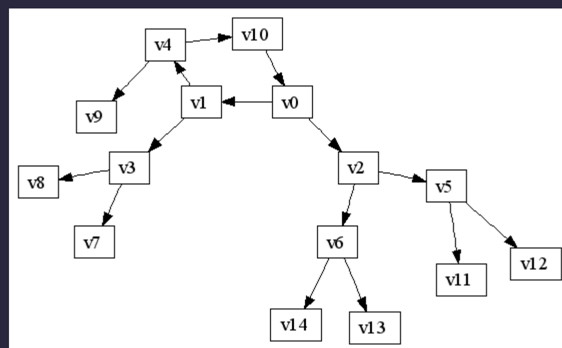
- y: node y-coordinates
- δ : edge direction (e.g., 1 for i→j, 0 for undirected)

- Says: If i points to j, it should have a lower y-value

Typical Sugiyama layout (dot)
- preserves tree structure



DiG-CoLa method
- preserves edge lengths



slide borrowed from Tim Dwyer

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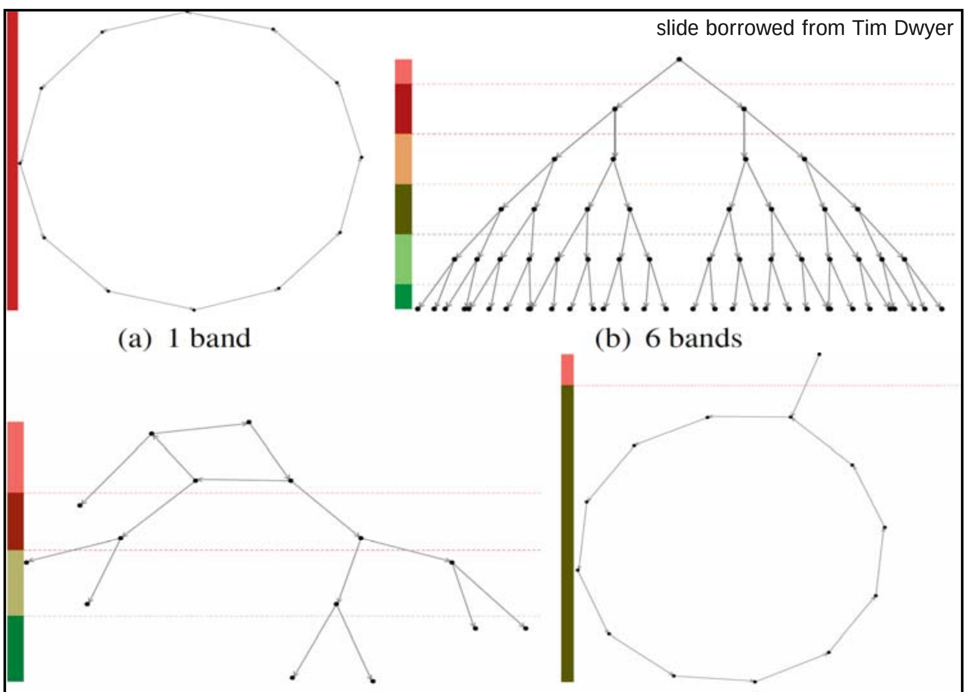


(a) 1 band

(b) 6 bands

(c) 4 bands

(d) 2 bands

Attribute-Driven Layout

Large node-link diagrams get messy!

What if the data has additional structure we can exploit for the layout?

Idea: Use data attributes to perform layout

- e.g., scatter plot based on node values

Both filtering and interaction (brushing) can be used to explore connectivity

Attribute-Driven Layout

The “Skitter” Layout

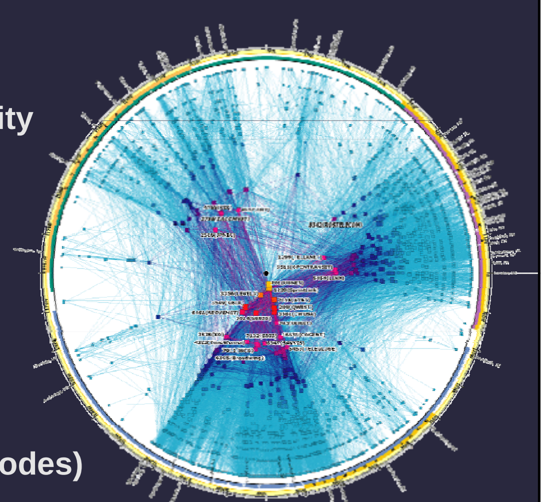
- Internet Connectivity
- Radial Scatterplot

Angle = Longitude

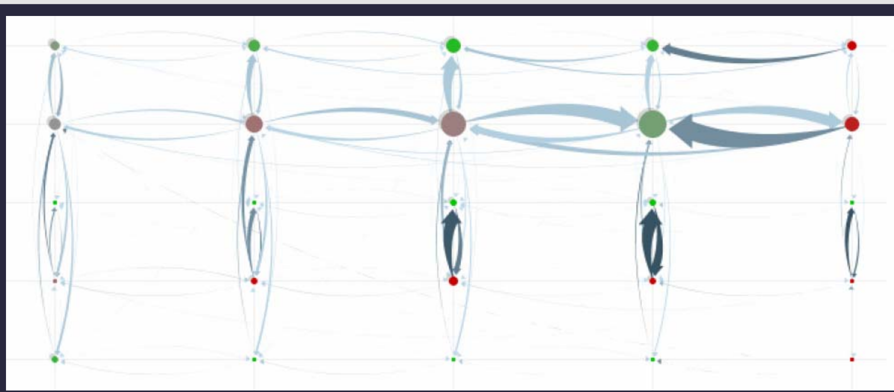
- geography

Radius = Degree

- # of connections
- (a statistic of the nodes)



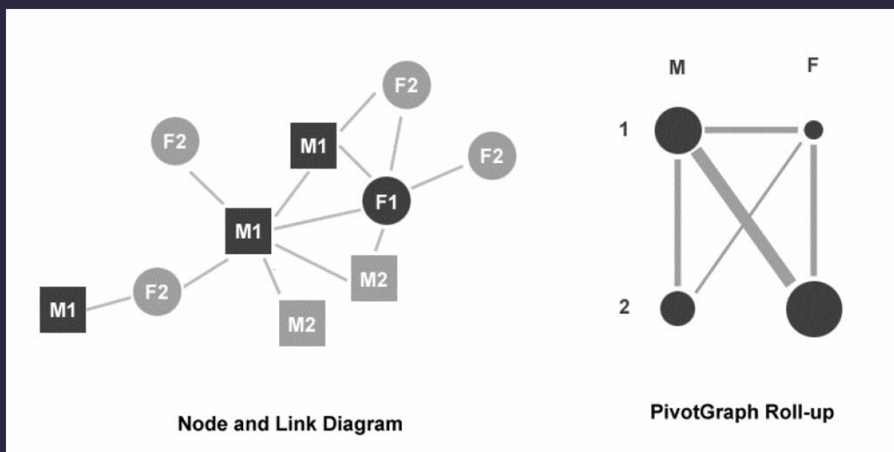
PivotGraph [Wattenberg 2006]



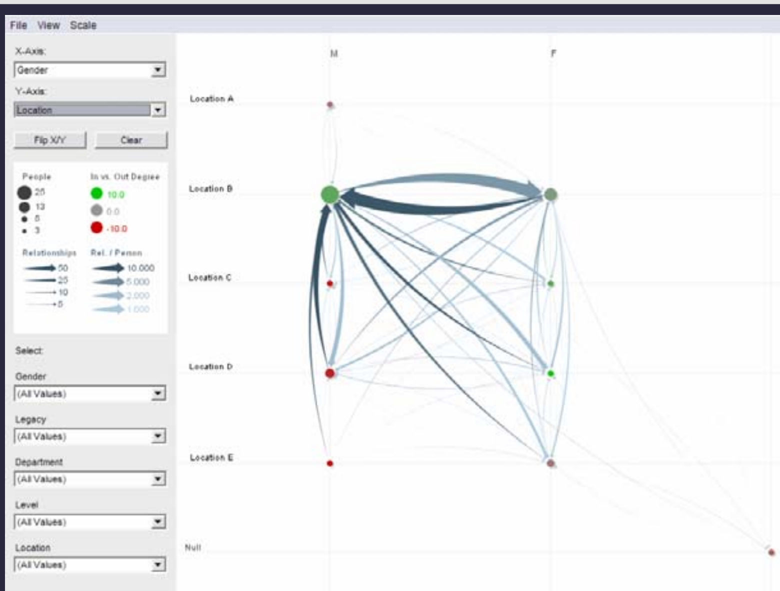
Tabular layout of aggregated graphs according to node data values.

Similar to pivot tables and Tableau.

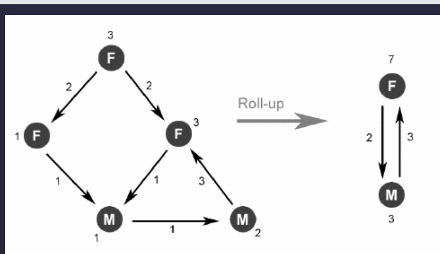
PivotGraph

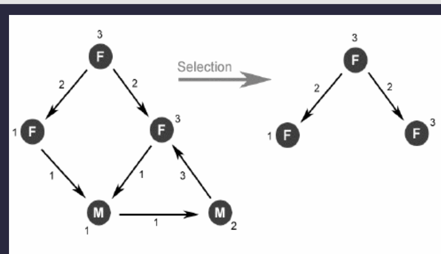


PivotGraph



Operators





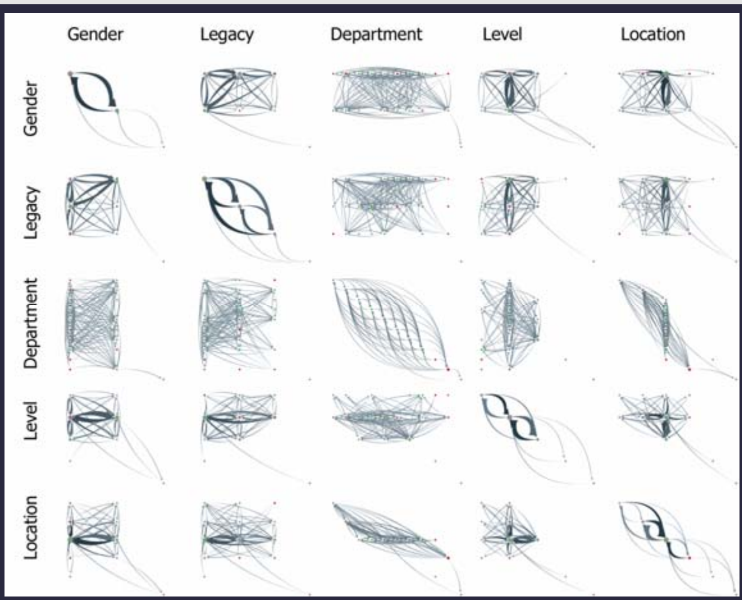
Roll-Up

Aggregate items with matching data values

Selection

Filter on data values

PivotGraph Matrices

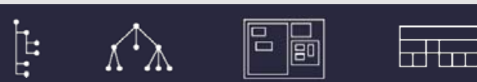


Limitations of PivotGraph

Only 2 variables (no nesting as in Tableau)
Doesn't support continuous variables
Multivariate edges?

Summary

Tree Layout



- Indented / Node-Link / Enclosure / Layers
- How to address issues of scale?
  - Filtering and Focus + Context techniques

Graph Layout

- Tree layout over spanning tree
- Hierarchical “Sugiyama” Layout
- Optimization Techniques
- Attribute-Driven Layout