CS-184: Computer Graphics

Lecture 18: Global Illumination

Maneesh Agrawala
University of California, Berkeley

Slides based on those of James O'Brien

Announcements

Assignment 5: due Fri Nov 5 by 11pm

Final Project: due ????

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Final Project: due ????

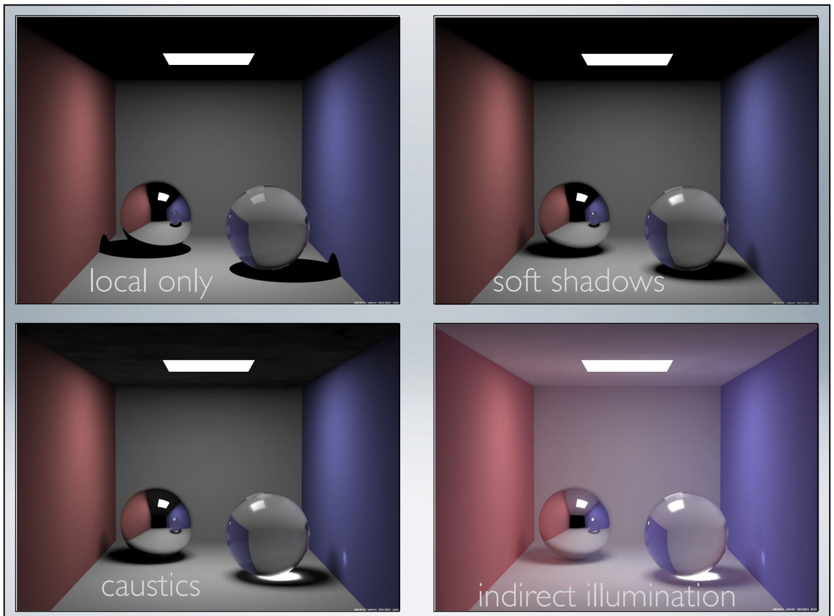
Today

The Rendering Equation

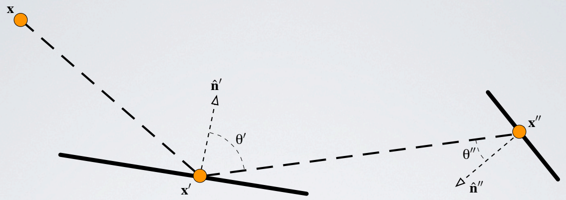
Radiosity Method

Photon Mapping

Ambient Occlusion

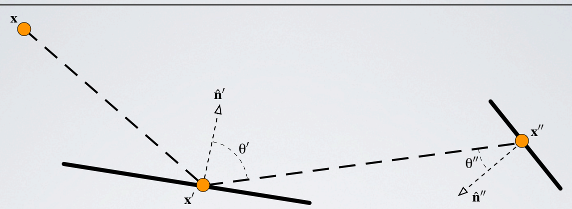


The Rendering Equation



The light shining on x from x' is equal to:
- The emitted light from x' toward x, plus
- For each bit of surface in the scene, how much light shines from that bit onto x' and is reflected toward x, scaled appropriately

The Rendering Equation



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$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'') L_s(\mathbf{x}',\mathbf{x}'') \frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2} \mathrm{d}\mathbf{x}''\right]$$

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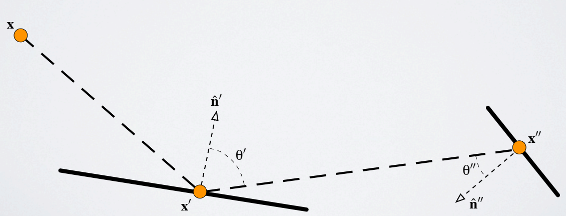

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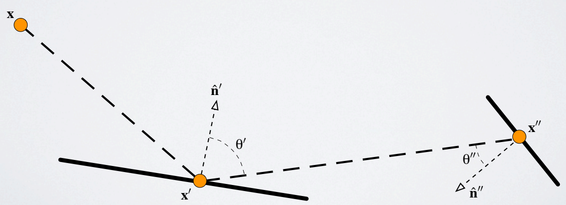
Light energy hitting x from x'



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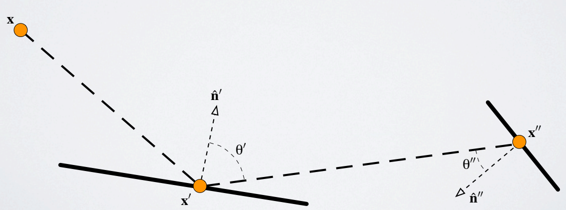

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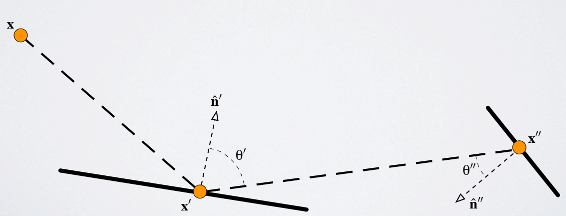
Can x see x' ?



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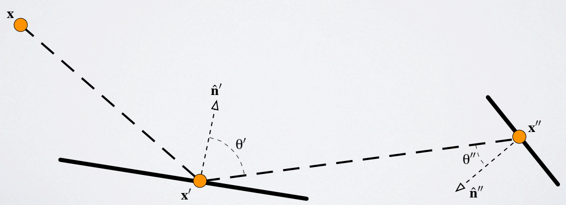

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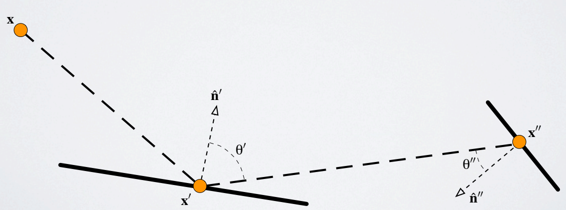
Light emitted from x' toward x



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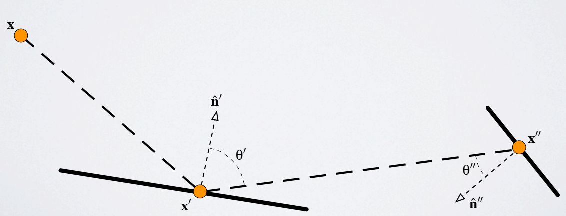

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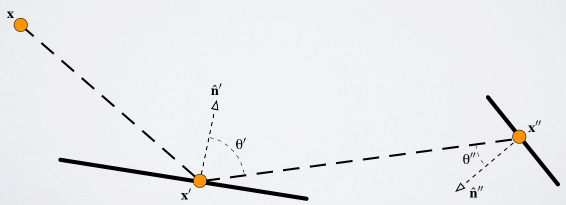
sum over every bit of surface in the scene



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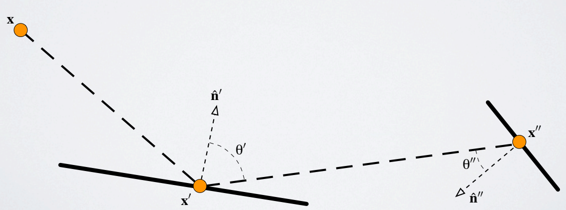

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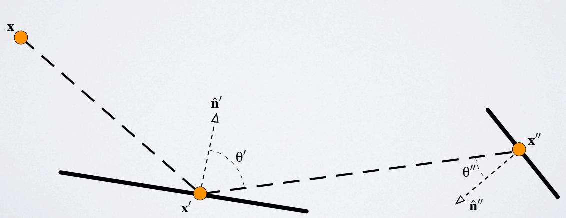
Light emitted from x'' toward x'



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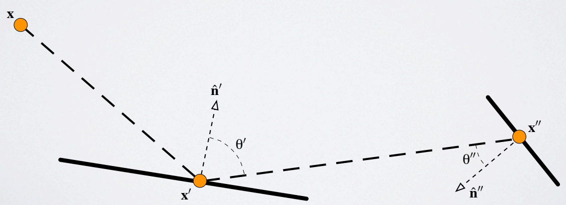

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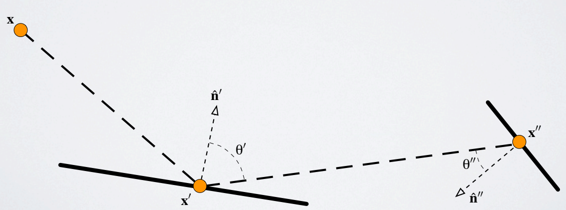
scaled down by the BRDF of x'



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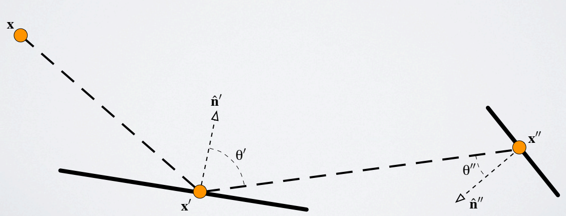

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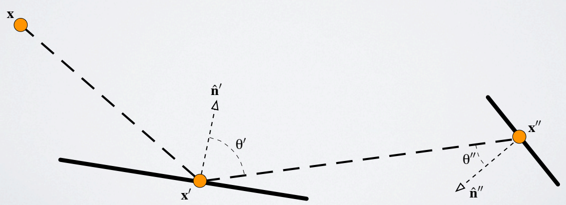
scaled down by distance and relative orientation ("form factor")



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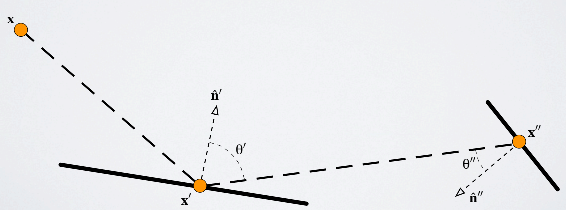


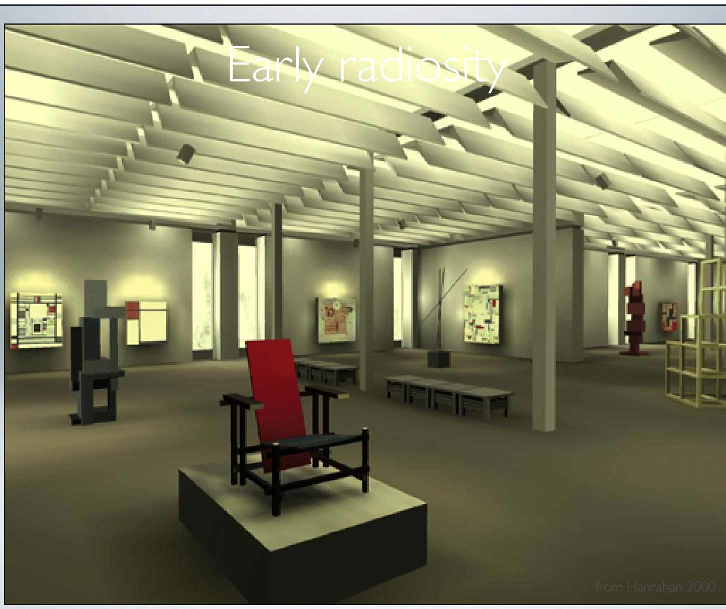
Radiosity

Assume all materials perfectly Lambertian (diffuse only, no specularities)

- Removes all dependance on directions
- Reduces dimensionality of lightfield
- Allows a FEM solution (break up into chunks)

Can also relax assumption slightly...

Early radiosity



from Hanrahan 2000

Assume Lambertian

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

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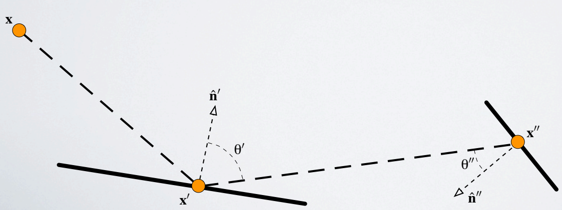

Assume Lambertian

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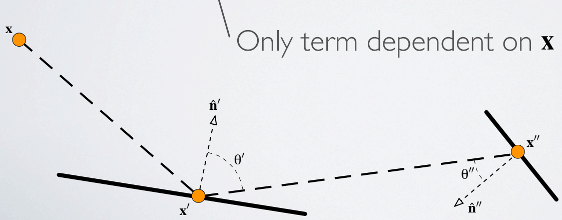
Only term dependent on $\mathbf{x}$

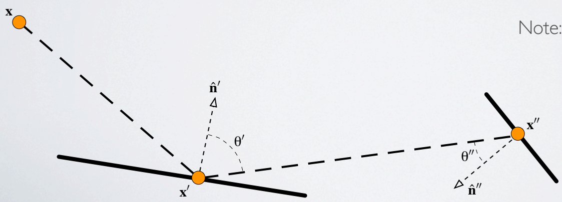


Rewrite in Terms of Radiosity

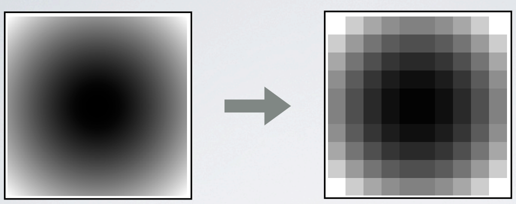
$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E_{x'} + \int_S \rho_{x'} L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

$$H_{x'} = E_{x'} + \rho_{x'}\int_S \delta(\mathbf{x}',\mathbf{x}'')\frac{H_{x''}}{2\pi}\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''$$

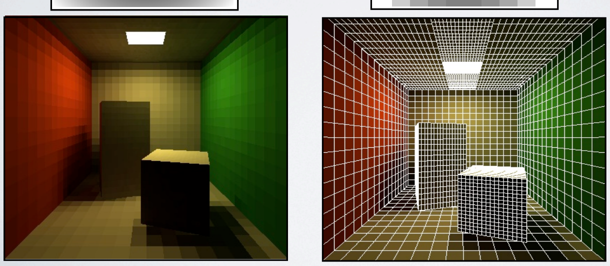
Note: we changed defn of E here.



Discretize into Patches

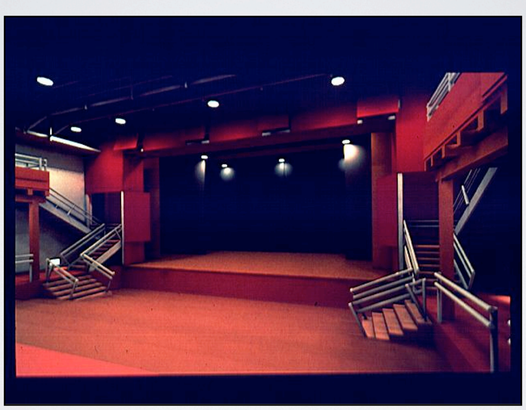


Piece-wise constant patches



Example mesh for Cornell Box
by Mark Schmelzenbach

Discretize into Patches



The Candlestick Theater,
Mark Mack Architects.

Discretize into Patches

The Candlestick Theater,
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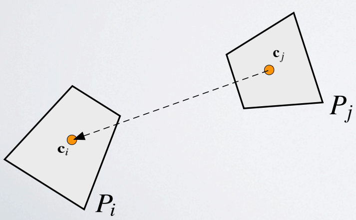
Rewrite in Terms of Patches

$$H_{x'} = E_{x'} + \rho_{x'} \int_S \delta(\mathbf{x}', \mathbf{x}'') \frac{H_{x''}}{2\pi} \frac{\cos(\theta') \cos(\theta'')}{||\mathbf{x}' - \mathbf{x}''||^2} d\mathbf{x}''$$

$$H_i = E_i + \rho_i \sum_j H_j \int_{S_j} \delta_{ij} \frac{\cos(\theta_i) \cos(\theta_j)}{2\pi ||\mathbf{c}_i - \mathbf{x}||^2} d\mathbf{x}$$



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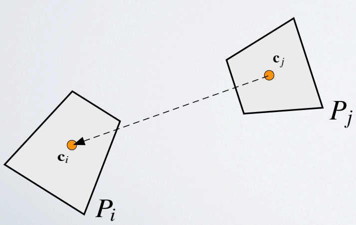
Form factor from ***j*** to ***i***, F_{ij}



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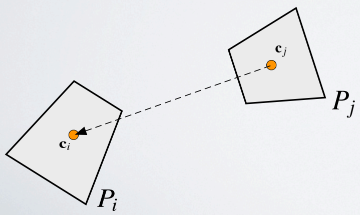
Form factor from *j* to *i*, F_{ij}



Example of a rough approximation:

$$F_{ij} \approx \delta_{ij} \frac{\cos(\theta_i)\cos(\theta_j)}{2\pi ||\mathbf{c}_i - \mathbf{c}_j||^2} A_j$$

Radiosity Method

Given the E_i and ρ_i

First compute F_{ij}

Then solve $H_i = E_i + \rho_i \sum_j H_j F_{ij}$

$$\mathbf{h} = \mathbf{e} + \mathbf{A}\mathbf{h}$$
$$(\mathbf{I} - \mathbf{A})\mathbf{h} = \mathbf{e}$$

Comments:

- The matrix $\mathbf{A}$ is typically very large
- It is also sparse (why?)
- Should be solved with an iterative method
  - e.g.: Jacobi or Gauss-Seidel
- **Solution is view independent**

Radiosity Method

Given the light emitted and surface properties

First compute F_{ij} form factors between patches

Then **solve a linear system to balance energy between all patches**

Comments:

- The system is very large
- It is also sparse (why?)
- Should be solved with an iterative method
  - e.g.: Jacobi or Gauss-Seidel
- **Solution is view independent**

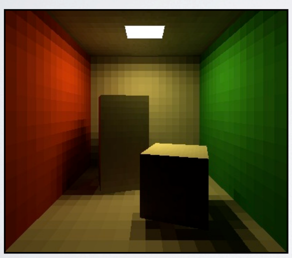
Progressive Radiosity

From dissertation "Efficient and predictive realistic image synthesis"
by Karol Myszkowski

Touchup

Each patch will have a constant color

- Smooth solution (**e.g.** average to vertices)



Example mesh for Cornell Box
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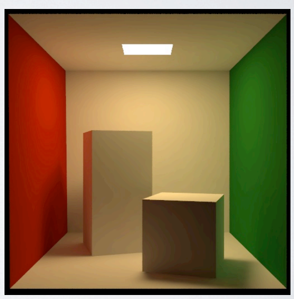


Does not match but you get the idea...

Other Things

Each patch will have a constant color

- Smooth solution (**e.g.** average to vertices)

No specular reflection

- Add Phong specular term or raytraced specular reflection

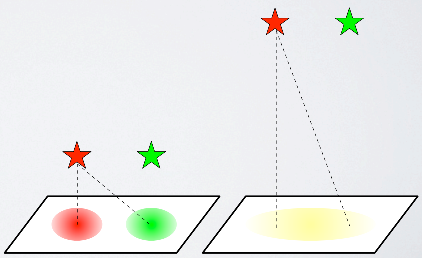
Grid artifacts

- Be clever with grid...

Hierarchical Radiosity

Light smoothes with distance

- Compare $1/h^2$ with $1/(h^2+d^2)$ as h gets large



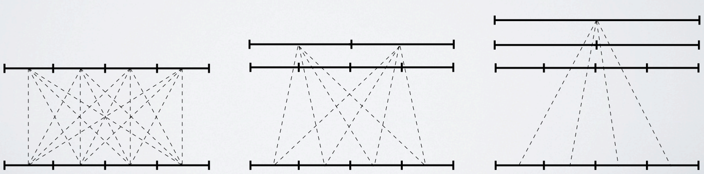
Hierarchical Radiosity

Light smoothes with distance

- Compare $1/h^2$ with $1/(h^2+d^2)$ as h gets large

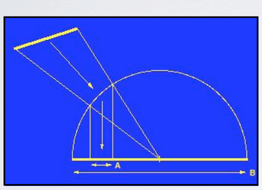
Group patches into hierarchy

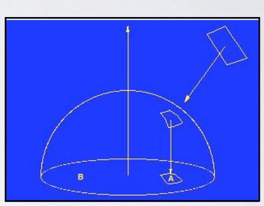
- Far interactions use lower-res form factors



Computing Form Factors

Form factors have a geometric meaning



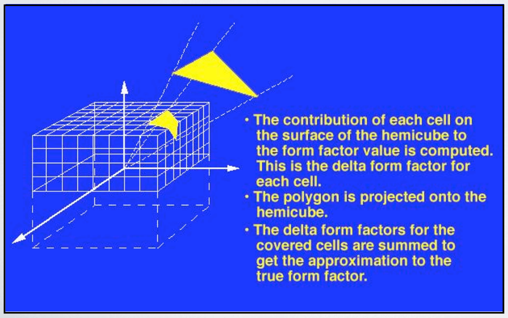


Images from
SIGGRAPH 93 Education Slide Set
by Stephen Spencer

Computing Form Factors

Form factors have a geometric meaning

"Hemicube" algorithm uses regular scan conversion



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Computing Form Factors

Form factors have a geometric meaning

"Hemicube" algorithm uses regular scan conversion

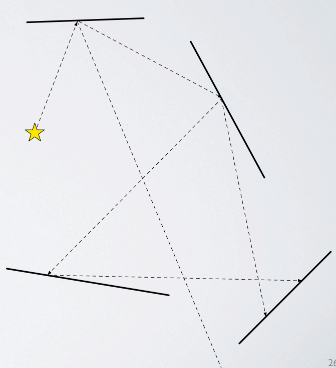
Also computed by ray-based sampling

In practice, computing form factors is the bottleneck

Photon Mapping

Lights cast "photons" into environment

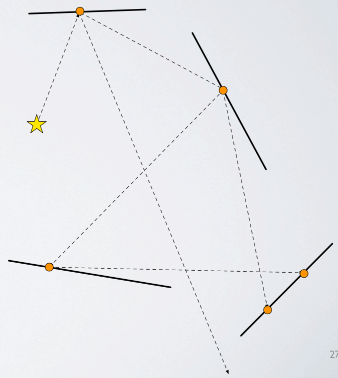
- Cast in random directions
- Trace into environment
- Store records at intersections



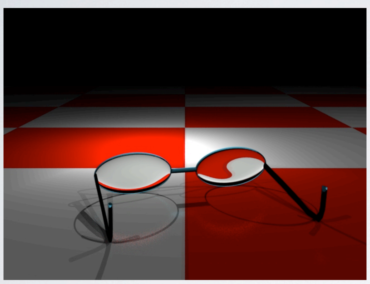
Photon Mapping

Lights cast "photons" into environment

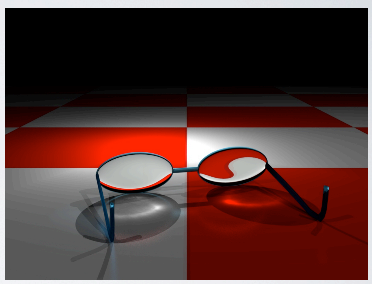
- Cast in random directions
- Trace into environment
- Store records at intersections
  - With KD-Trees...



Comparison



Ray Tracing



Ray Tracing w/ Photon Map

Catherine Bendebury and Jonathan Michaels
CS 184 Spring 2005

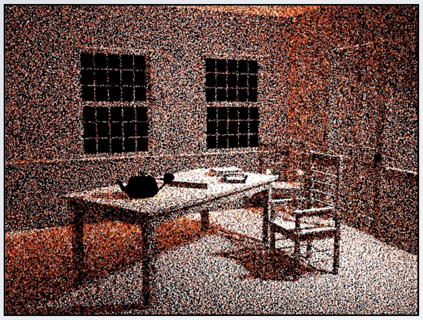
Photon Mapping

Image by Per Christensen

A ray traced image

Note:
Dark shadows
Unlit corners
Nice reflections

Photon Mapping



Raw photons

Note:
- Noisy
- Sparse

Image by Per Christensen

Photon Mapping

Interpolated Photons

Note:
- Still noisy
- Biased

Image by Per Christensen

Photon Mapping

Interpolated Photons (multiplied by diffuse)

Note:
- Still noisy
- Biased

Image by Per Christensen

Photon Mapping

Final Gather

- Ray trace scene
- Direct and specular rays as normal
- Diffuse rays traced into photon map

- ***Diffuse reflection smoothes noise***

Photon Mapping

Image by Per Christensen

Final Image

Note:
  Not noisy
  Nice lighting
  Reflections
  May still be biased

  Final gather often bottleneck...

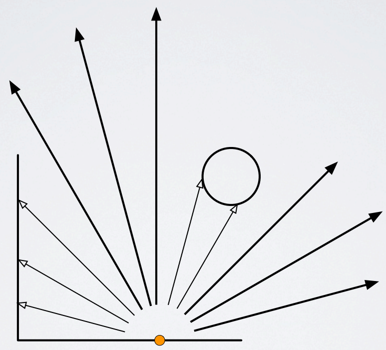
Ambient Occlusion

A "hack" to create more realistic ambient illumination cheaply

Assume light from everywhere is partially blocked by local objects

- At a point on the surface cast rays at random
- Ambient term is proportional to percent of rays that hit nothing
- Weight average by cosine of angle with normal
- Take into account how far before occluded

Ambient Occlusion



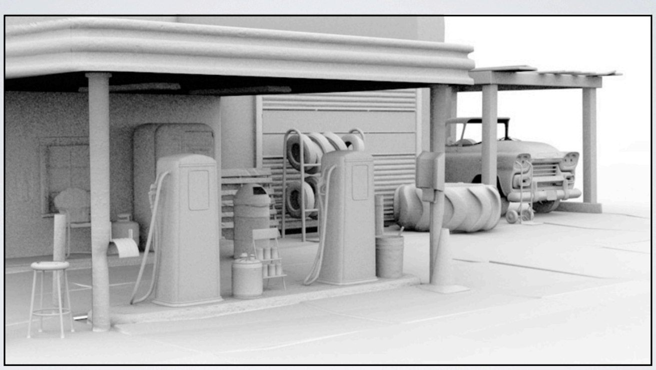
Ambient Occlusion

Diffuse Only

Ambient Occlusion

Combined

Ambient Occlusion



nVidia Gelato Demo Image