# CS-184: Computer Graphics

## Lecture 17: Radiometry

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# Today

Radiometry: measuring light

- Local Illumination and Raytracing were discussed in an ***ad hoc*** fashion
- Proper discussion requires proper units
- Not just pretty pictures... but correct pictures



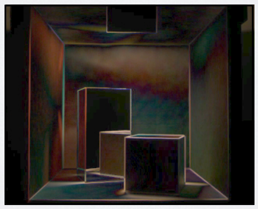
# Matching Reality

Unknown

# Matching Reality

Photo

Rendered



Cornell Box Comparison
Cornell Program of Computer Graphics



# Units

Light energy

- Really power not energy is what we measure
- Joules / second ( J/s ) = Watts ( W )

Spectral energy density

- Power per unit spectrum interval
- Watts / nano-meter ( W/nm )
- Properly done as function over spectrum
- Often just sampled for RGB

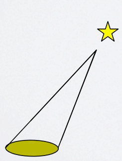
Often we assume people know we're talking about S.E.D. and just say E...

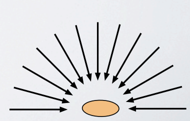


# Irradiance

Total light striking surface from all directions

- Only meaningful w.r.t. a surface
- Power per square meter ( $\mathrm{W/m^2}$)
- Really S.E.D. per square meter ( $\mathrm{W/m^2\ /nm}$ )
- Not all directions sum the same because of foreshortening

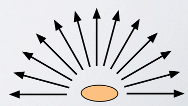




# Radiant Exitance

Total light ***leaving*** surface over all directions

- Only meaningful w.r.t. a surface
- Power per square meter ($\mathrm{W/m^2}$)
- Really S.E.D. per square meter ($\mathrm{W/m^2\ /nm}$)
- Also called Radiosity
- Sum over all directions $\not\Rightarrow$ same in all directions





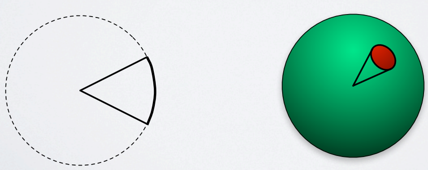
# Solid Angles

Regular angles measured in ***radians*** $[0..2\pi]$

- Measured by arc-length on unit circle

Solid angles measured in ***steradians*** $[0..4\pi]$

- Measured by area on unit sphere
- Not necessarily little round pieces...





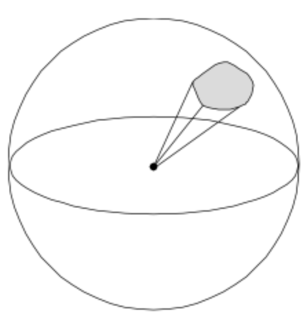
## Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

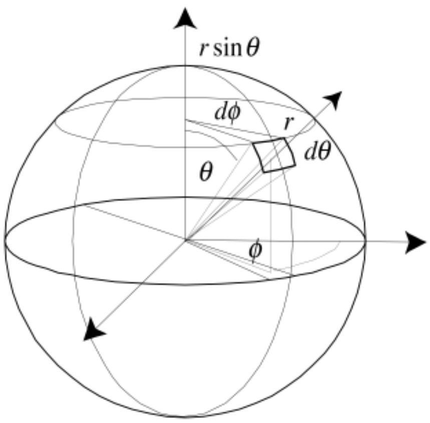
  $\Rightarrow$ **circle has $2\pi$ radians**

- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ **sphere has $4\pi$ steradians**



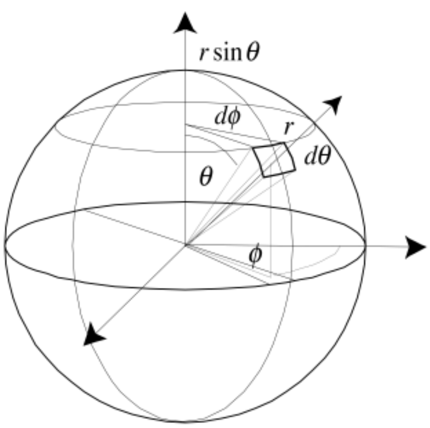
## Differential Solid Angles




$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2\sin\theta\,d\theta\,d\phi$$



## Differential Solid Angles

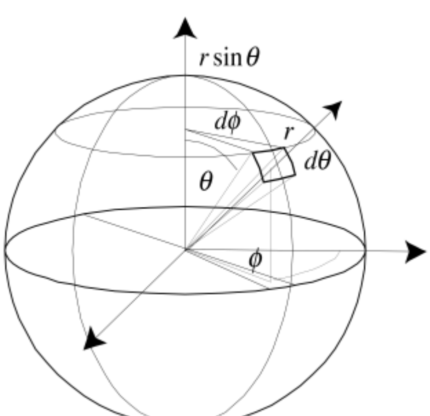



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2\sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$



## Differential Solid Angles




**Sphere** $S^2$

$$d\omega = \sin\theta\,d\theta\,d\phi$$

$$\Omega = \int_{S^2} d\omega$$
$$= \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi$$
$$= \int_{-1}^{1}\int_0^{2\pi} d\cos\theta\;d\phi$$
$$= 4\pi$$

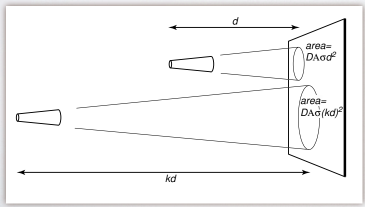
# Radiance

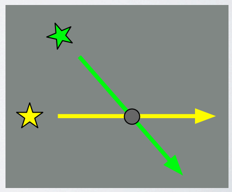
Light energy passing though a point in space within a given solid angle

- Energy per steradian per square meter ($\mathrm{W/m^2/sr}$)
- S.E.D. per steradian per square meter ($\mathrm{W/m^2/sr/nm}$)

Constant along straight lines in free space

- Area of surface being sampled is proportional to distance and light inversely proportional to squared distance



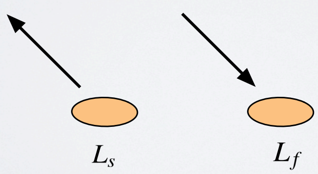





# Radiance

Near surfaces, differentiate between

- Radiance from the surface ( surface radiance )
- Radiance from other things ( field radiance )





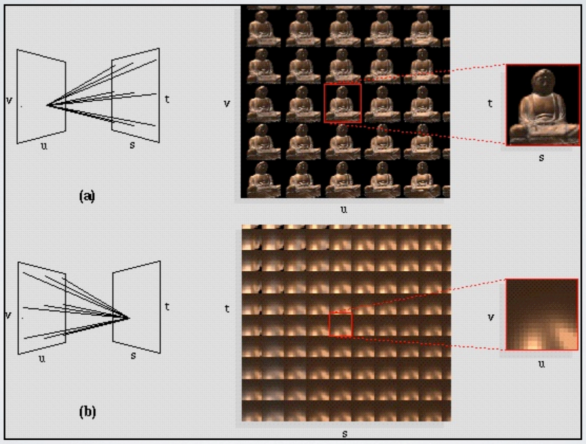

# Light Fields

Radiance at every point in space, direction, and frequency: 6D function

Collapse frequency to RGB, and assume free space: 4D function

Sample and record it over some volume

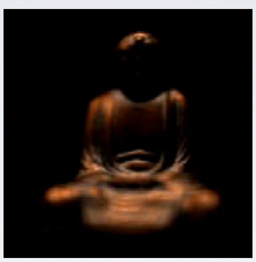
# Light Fields




Levoy and Hanrahan, SIGGRAPH 1996



# Light Fields



Levoy and Hanrahan, SIGGRAPH 1996



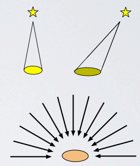
# Light Fields

Michelangelo's ***Statue of Night***
From the Digital Michelangelo Project

# Computing Irradiance

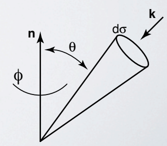
Integrate incoming radiance (field radiance) over all direction

- Take into account foreshortening



$$H = \int_\Omega L_f(\mathbf{k}) \cos(\theta) \mathrm{d}\sigma$$

$$H = \int_0^{2\pi} \int_0^{\pi/2} L_f(\theta, \phi) \cos(\theta) \sin(\theta)\, \mathrm{d}\theta\, \mathrm{d}\phi$$



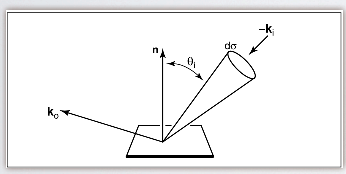



# Revisiting The BRDF

How much light from direction $k_i$ goes out in direction $k_o$

Now we can talk about units:

- BRDF is ratio of surface radiance to the foreshortened field radiance multiplied by small solid angle (denom. similar to integrand of irradiance - see prev. slide)




$$\rho(\mathbf{k}_i, \mathbf{k}_o) = \frac{L_s(\mathbf{k}_o)}{L_f(\mathbf{k}_i) \cos(\theta_i) \Delta\sigma_i}$$

We left out frequency dependance here...

Also note for perfect Lambertian reflector with constant BRDF $\rho = 1/\pi$



# The Rendering Equation

Total light going out in some direction is given by an integral over all incoming directions:

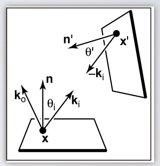
$$L_s(\mathbf{k}_o) = \int_\Omega \rho(\mathbf{k}_i, \mathbf{k}_o) L_f(\mathbf{k}_i) \cos(\theta_i) \mathrm{d}\sigma_i$$

- Note, this is recursive ( my $L_f$ is another's $L_s$)

# The Rendering Equation

$$L_s(\mathbf{k}_o) = \int_\Omega \rho(\mathbf{k}_i, \mathbf{k}_o) L_f(\mathbf{k}_i) \cos(\theta_i) \mathrm{d}\sigma_i$$

Rewrite explicitly in terms of surface radiances only




$$L_f(\mathbf{k}_i) = L_s(-\mathbf{k}_i) \qquad \Delta\sigma_i = \frac{\Delta A' \cos(\theta')}{||\mathbf{x} - \mathbf{x}'||^2}$$

$$L_s(\mathbf{x}, \mathbf{k}_o) = \int_{x' \text{ visible to } x} \frac{\rho(\mathbf{k}_i, \mathbf{k}_o) L_s(\mathbf{x}', \mathbf{x} - \mathbf{x}') \cos(\theta_i) \cos(\theta')}{||\mathbf{x} - \mathbf{x}'||^2} \mathrm{d}\mathbf{A}'$$

$$L_s(\mathbf{x}, \mathbf{k}_o) = \int_{\text{all } x'} \frac{\rho(\mathbf{k}_i, \mathbf{k}_o) L_s(\mathbf{x}', \mathbf{x} - \mathbf{x}') \delta(\mathbf{x}, \mathbf{x}') \cos(\theta_i) \cos(\theta')}{||\mathbf{x} - \mathbf{x}'||^2} \mathrm{d}\mathbf{A}'$$

$$\delta(\mathbf{x}, \mathbf{x}') = \begin{cases} 1 \text{ if } \mathbf{x} \text{ and } \mathbf{x}' \text{ are mutually visible} \\ 0 \text{ otherwise} \end{cases}$$

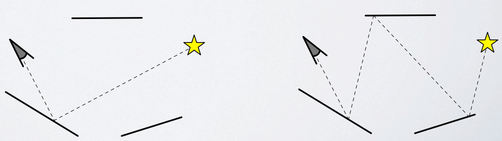


# Light Paths

Many paths from light to eye

Characterize by the types of bounces

- Begin at light
- End at eye
- "Specular" bounces
- "Diffuse" bounces





# Light Paths

Describe paths using strings

- LDE, LDSE, LSE, ***etc.***

Describe types of paths with regular expressions

- L{D|S}*E ← Visible paths
- L{D|S}S*E ← Standard raytracing
- L{D|S}E ← Local illumination
- LD*E ← Radiosity method (have not talked about yet)