Announcements

Assignment 5: due Fri Nov 5 by 11pm
Assigning Texture Coordinates
Distortion
Antialiasing
Bump & Displacement Maps, Environment Maps, Shadow Maps
Procedural Textures

Texture Mapping
Objects have properties that vary across the surface

So we make the shading parameters vary across the surface
Texture Mapping

Texture mapping: a technique of defining surface properties (especially shading parameters) in such a way that they vary as a function of position on the surface.
Examples

Wood gym floor with smooth finish
- diffuse color $k_D$ varies with position
- specular properties $k_S, n$ are constant

Glazed pot with finger prints
- diffuse and specular colors $k_D, k_S$ are constant
- specular exponent $n$ varies with position

Adding dirt to painted surfaces
Simulating stone, fabric, …
- to approximate effects of small-scale geometry

More than Diffuse Color

Use a 2D image and map it to the surface of an object

Mapping textures to surfaces

Usually the texture is an image (function of $u, v$)
- Big question: where on the surface does the image go?
- Obvious only for a flat rectangle the same shape as the image
- Otherwise more interesting

Note that 3D textures also exist
- Texture is a function of $(u, v, w)$
- Can just evaluate texture at 3D surface point
- Good for solid materials
- Often defined procedurally
Assigning Texture Coordinates

Parameterization

Q: How do we decide where on the geometry each color from the image should go?

Mapping Textures to Surfaces

“Putting the image on the surface”

- Need a function $f$ that tells where each point on the image goes
- Similar to parametric surface function
- For parametric surfaces you get $f$ for free
Texture Coordinate Functions

Non-parametrically defined surfaces: more to do

- Can’t assign texture coordinates as we generate the surface
- Need inverse of the function $f$

Texture coordinate fn.

$\phi : S \rightarrow \mathbb{R}^2$

- For a vertex at $p$
  - get texture at $\phi(p)$

Texture Coordinate Functions

Define texture image as a function

$$T : D \rightarrow C$$

- Where $C$ is the set of colors for the diffuse component

Diffuse color (for example) at point $p$ is then

$$k_D(p) = T(\phi(p))$$

Texture Coordinate Functions

Mapping from $S$ to $D$ can be many-to-one

- Every surface point gets only one color assigned
- OK (and in fact useful) for multiple surface points to be mapped to the same texture point
  - e.g. repeating tiles
Repeating Textures

Image Tiles allow repeating textures

- Images must be manipulated to allow tiling
- Often result in visible artifacts
  - There are methods to get around artifacts...

Examples

[Paul Bourke]
Planar mapping
Like projections, drop z coord \((u,v) = (x,y)\)
Problems: what happens near \(z = 0\)?

Cylindrical Mapping
Cylinder: \(r, \theta, z\) with \((u,v) = (\theta/(2\pi), z)\)
- Note seams when wrapping around \((\theta = 0\ or\ 2\pi)\)

Spherical Mapping
Convert to spherical coordinates: use latitude/long,
- Singularities at north and south poles
Sphere Mapping
For a sphere: latitude-longitude coordinates

- $\phi$ maps point to its latitude and longitude

Examples of Coordinate Functions
Non-parametric surfaces: project to parametric surface

Examples: Parametric Surface
A parametric surface (e.g. spline patch)
- Surface parameterization gives mapping function directly (well, the inverse of the parameterization)
Texture coordinates become per-vertex data
- Can think of them as 2nd position
  - Each vertex has a position in 3D space and in 2D texture space

How to come up with vertex \((u,v)\)s?
- Use any or all of the methods just discussed
  - In practice this is how you implement those for curved surfaces approximated with triangles
- Alternatively: Use some kind of optimization
  - Try to choose vertex \((u,v)\)s to result in a smooth, low distortion map
Examples: Triangle

Triangles
- specify \((u,v)\) for each vertex
- define \((u,v)\) for interior by barycentric (linear) interpolation

\[
\begin{align*}
\mathbf{p}(\beta, \gamma) &= \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \\
u(\beta, \gamma) &= u_a + \beta(u_b - u_a) + \gamma(u_c - u_a) \\
v(\beta, \gamma) &= v_a + \beta(v_b - v_a) + \gamma(v_c - v_a)
\end{align*}
\]

Dealing with Distortion

Can’t Linearly Interpolate Tex. Coords.

Triangle Scan Conversion
For each pixel:
1. Interpolate \(u\) & \(v\) down edges, across spans
2. Look up nearest texel in texture map
3. Color pixel according to texel color (possibly modulated by lighting calculations)

\[
\begin{align*}
\mathbf{p}(\beta, \gamma) &= \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \\
u(\beta, \gamma) &= u_a + \beta(u_b - u_a) + \gamma(u_c - u_a) \\
v(\beta, \gamma) &= v_a + \beta(v_b - v_a) + \gamma(v_c - v_a)
\end{align*}
\]

Leads to Artifacts: [http://graphics.cs.mit.edu/classes/6.837/F98/Lecture21/Slide05.html](http://graphics.cs.mit.edu/classes/6.837/F98/Lecture21/Slide05.html)
Naïve Texturing Artifacts

Warping at edges of triangles

A more obvious example:
http://graphics.lcs.mit.edu/classes/6.837/F98/Lecture21/Slide06.html

Consider the geometry of interpolating parameters more carefully

Interpolating Parameters

Problem due to interpolating parameters in screen-space

Uniform steps in screen space ≠ uniform steps in world space

Texture Mapping

Linear interpolation of texture coordinates
Correct interpolation with perspective divide

Hill Figure 8.42
Interpolating Parameters

Perspective foreshortening not being applied to interpolated parameters

- Parameters should be compressed with distance
- Linearly interpolating them in screen-space doesn’t do this

Is this a problem with Gouraud shading?

- A: It can be, but we usually don’t notice (why?)
  http://graphics.cs.mit.edu/classes/6.837/F98/Lecture21/Side17.html

Perspective-Correct Interpolation

Skipping a bit of math to make a long story short…

- Rather than interpolating $u$ and $v$ directly, interpolate $u/z$ and $v/z$
  These do interpolate correctly in screen space
  Also need to interpolate $z$ and divide per-pixel

Problem: may not know $z$ (didn’t need it for rasterizing)

- Solution: we do know $w \propto z/n$
- So…interpolate $w$, $u/w$ and $v/w$ (interpolation of $w$ is non-lin. - see book)

Unfortunately involves a divide per pixel


Depth Distortion

Recall depth distortion from perspective

- Interpolating in screen space different than in world
- Ok, for shading (mostly)
- Bad for texture

World

Half way in screen space

Screen

Half way in world space
We know the $s_i$, $P_i$, and $b_i$, but not the $a_i$.
Depth Distortion

\[ S_1 = P_1 / h_1 \]

\[ S_2 = P_2 / h_2 \]

\[ S_3 = P_3 / h_3 \]

\[ S_4 = P_4 / h_4 \]

\[ X = \sum_i S_i b_i \]

\[ \sum_i S_i b_i = \left( \sum_i P_i a_i \right) / \left( \sum_j h_j a_j \right) \]

Depth Distortion

\[ S_1 = P_1 / h_1 \]

\[ S_2 = P_2 / h_2 \]

\[ S_3 = P_3 / h_3 \]

\[ S_4 = P_4 / h_4 \]

\[ X = \sum_i S_i b_i \]

\[ \sum_i P_i b_i / h_i = \left( \sum_i P_i a_i \right) / \left( \sum_j h_j a_j \right) \]

Formula for \( a_i \) should be independent of given vertex locations \( P_i \)

\[ b_i / h_i = a_i / \left( \sum_j h_j a_j \right) \quad \forall i \]
Depth Distortion

For a line: $a_i = \frac{h_2 b_i}{(b_1 h_2 + h_1 b_2)}$

For a triangle: $a_i = \frac{h_2 h_3 b_i}{(h_2 h_3 b_1 + h_1 h_3 b_2 + h_1 h_2 b_3)}$

Obvious Permutations for other coefficients.

Linear equations in the $a_i$.

\begin{align*}
\sum_j h_j a_j - b_i &= 0 \quad \forall i \\
\sum_i a_i &= \sum_i b_i = 1
\end{align*}
Antialiasing

Texture Filtering

Must sample texture to determine color at each pixel in image

Size of filter depends on projective warp

- Can prefilter images
  - Mip maps
  - Summed area tables

Angel Figure 9.4

Angel Figure 9.14
Mip Maps

Keep textures prefiltered at multiple resolutions

- For each pixel, linearly interpolate between two closest levels (e.g., trilinear filtering)
- Fast, easy for hardware

MIP-map Example

No filtering:

AAAAAAAGH
MY EYES ARE BURNING

MIP-map texturing:

Where are my glasses?

Summed-area tables

At each texel keep sum of all values down & right

- To compute sum of all values within a rectangle, simply subtract two entries
- Better ability to capture very oblique projections
- But, cannot store values in a single byte
Summed-Area Tables

Mipmaps assume each pixel projects to a square in texture (is a lie).
SAT can integrate texels covered by the pixel more exactly (still quickly).

Uses of Textures

Texture Mapping Variations

Can modulate any parameter in the Phong shading equation.
Non-Color Textures

Bump Mapping

Texture = change in surface normal!

Images by Paul Baker
www.paulprojects.net
Image Gradients as Bump Maps

Modify surface normal using gradients of texture image

\[ \nabla I(u, v) = \left( \frac{\partial I(u, v)}{\partial u}, \frac{\partial I(u, v)}{\partial v} \right) = (I_u, I_v) \]

Bump Mapping

Bump map: \( I(u, v) \) contains offsets to normal for each (u,v) pt.  
Orig. surface pt, normal and param: \( p, n, S(u, v) \)  \( n = S_u \times S_v \)
New surface pt and normal: normal: \( n' = n + \frac{I_u(n \times S_v)}{|n|} + \frac{I_v(n \times S_u)}{|n|} \)

More Bump Mapping

How can you tell a bumped-mapped object from an object in which the geometry is explicitly modeled?

Displacement Maps

Move geometry based on texture map

- Expensive and difficult to implement in many rendering systems
- Note silhouette
Displacement Mapping

Illumination Maps

Quake introduced *illumination maps* or *light maps* to capture lighting effects in video games

Texture map: ![Texture map example]

Light map: ![Light map example]

Texture map + light map: ![Texture map + light map example]
Environment Maps

Environment maps allow crude reflections

Treat object as infinitesimal

- Reflection only based on surface normal

Errors hard to notice for non-flat objects

Environment Maps

Images from *Illumination and Reflection Maps: Simulated Objects in Simulated and Real Environments*
Gene Miller and C. Robert Hoffman
SIGGRAPH 1984 "Advanced Computer Graphics Animation" Course Notes

Environment Maps

Sphere based parameterization

- Wide angle image or
- Photo of a silver ball

Images by Paul Haeberli
Environment Mapping

From ray tracing we know what we’d like to compute

- Trace a recursive ray into the scene—too expensive

If scene is infinitely far away, depends only on direction

- A two-dimensional function

Environment Map

A function from the sphere to colors, stored as a texture

Spherical Environment Map

Convert reflection ray direction to point in map
Environment Maps
Cube based parameterization (see book)

- Used in 1985 in movie *Interface*
- Effect by group from the New York Institute of Technology
Environment Maps

Used in 1985 in movie Interface
  • Effect by group from the New York Institute of Technology

Shadow Maps

Pre-render scene from perspective of light source
  • Only render Z-Buffer (the shadow buffer)

Render scene from camera perspective
  • Compare with shadow buffer
  • If nearer light, if further shadow

Shadow Maps

Shadow Buffer

Image w/ Shadows

From Stamminger and Drettakis
SIGGRAPH 2002

Note: These images don't really go together see the paper...
Deep Shadow Maps

Some objects only partially occlude light

- A single shadow value will not work
- Similar to transparency in Z-Buffer

From Lokovic and Veach
SIGGRAPH 2000

Procedural Textures

Generate texture based on some function

- Well suited for “random” textures
- Often modulate some noise function
Solid Textures

Texture values indexed by 3D location
Expensive storage
Compute on the fly,
e.g. Perlin noise

Procedural Texture

Procedural Texture Gallery