

CS-184: Computer Graphics

Lecture 14: Subdivision

Maneesh Agrawala
University of California, Berkeley

Slides based on those of James O'Brien

Subdivision

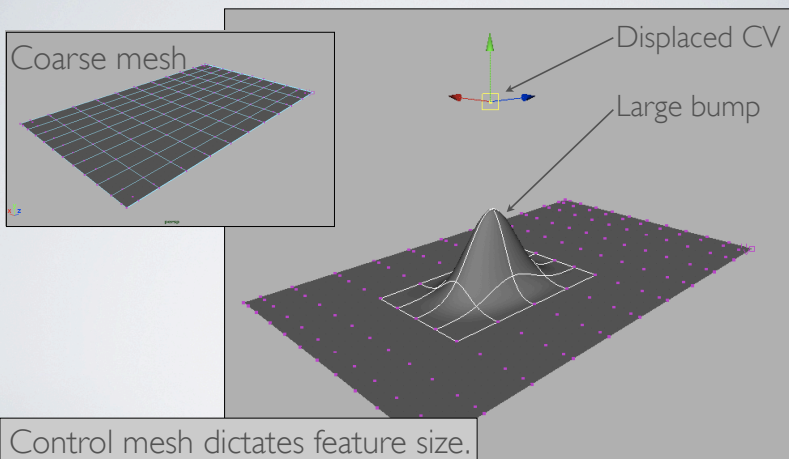
Start with:

- Given control points for a curve or surface, find new control points for a sub-section of curve/surface

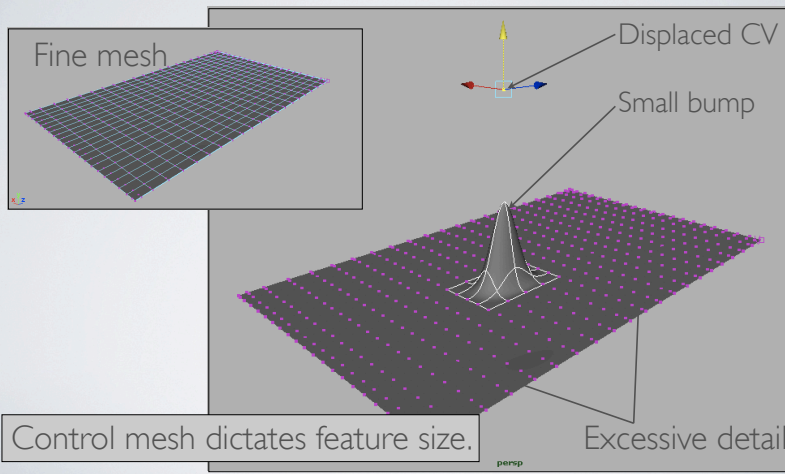
Key extension to basic idea:

- Generalize to non-regular surfaces

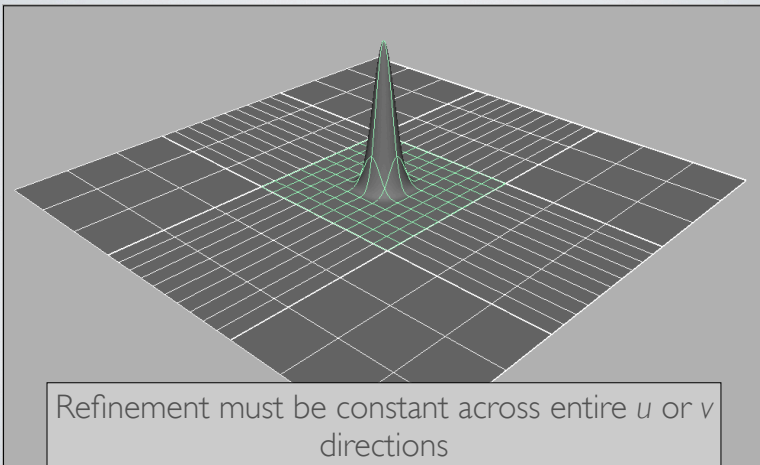
Consider NURBS Surface



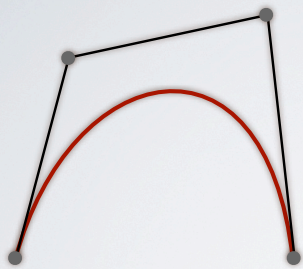
Consider NURBS Surface



Tensor Product Surface Refinement



Bézier Subdivision



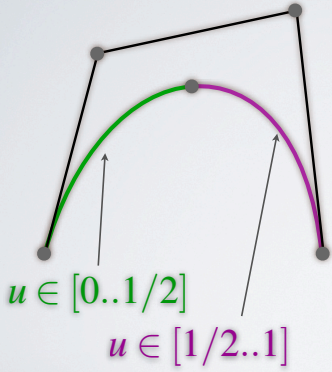
$$\mathbf{x}(u) = \sum_i b_i(u) \mathbf{p}_i$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}$$

Vector of control points

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

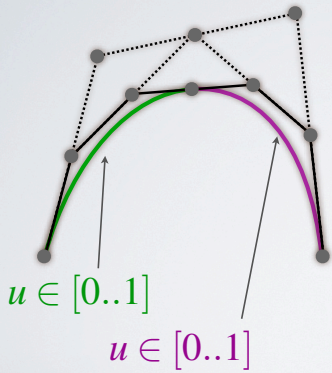
Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}$$

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Bézier Subdivision

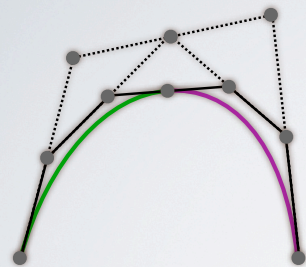


$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P}$$

Can't change these....

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{P} \quad u \in [0, \frac{1}{2}]$$

$$\mathbf{x}(u) = [1, \frac{u}{2}, \frac{u^2}{4}, \frac{u^3}{8}] \beta_Z \mathbf{P} \quad u \in [0, 1]$$

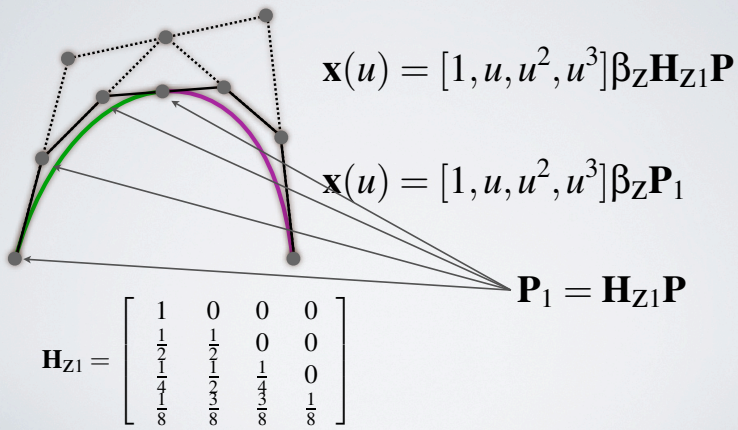
$$\mathbf{x}(u) = [1, u, u^2, u^3] \mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/8 \end{bmatrix}$$

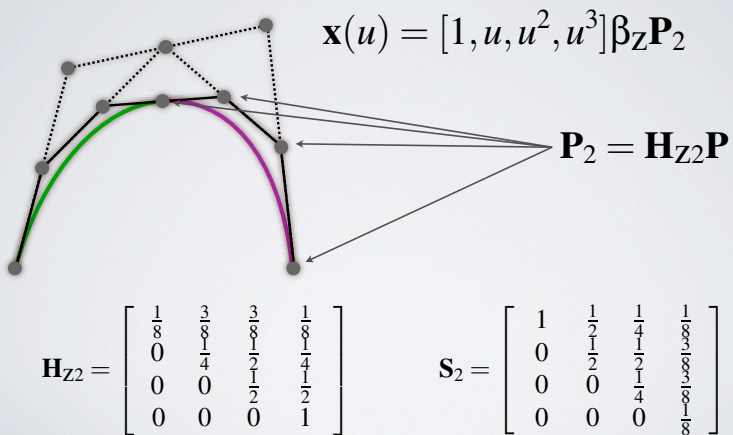
$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \beta_Z^{-1} \mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3] \beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

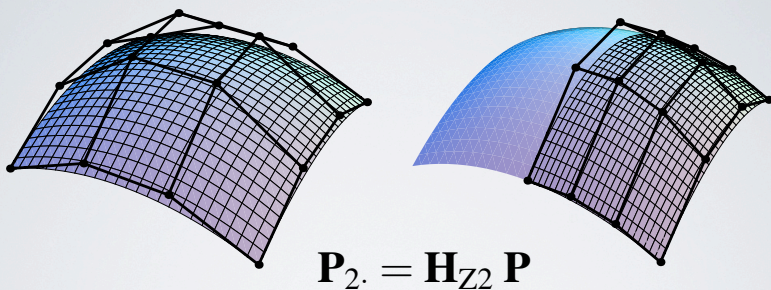
Bézier Subdivision



Bézier Subdivision



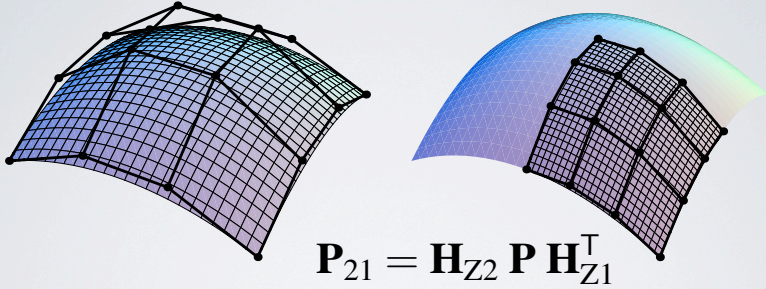
Bézier Subdivision



$$\mathbf{x}(u, v) = [1, u, u^2, u^3] \beta_Z \mathbf{P} \beta_Z^T [1, v, v^2, v^3]^T$$

4 x 4 matrix of control points \longrightarrow

Bézier Subdivision



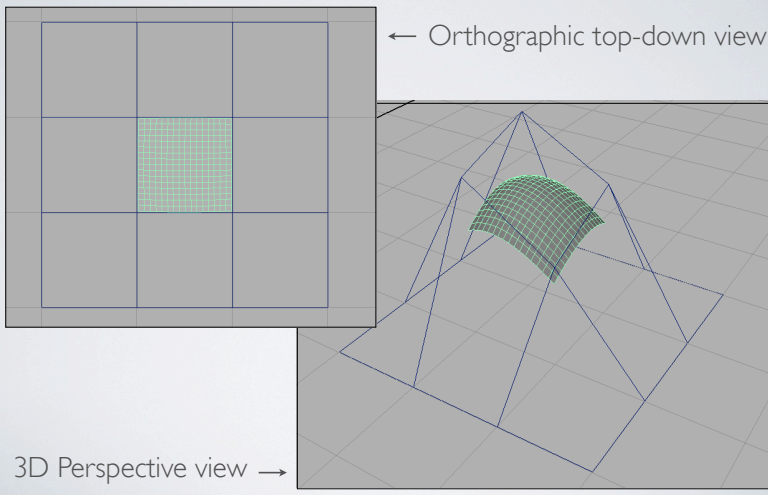
$$\mathbf{P}_{21} = \mathbf{H}_{Z2} \mathbf{P} \mathbf{H}_{Z1}^T$$

$$\mathbf{x}(u, v) = [1, u, u^2, u^3] \beta_Z \mathbf{P} \beta_Z^T [1, v, v^2, v^3]^T$$

4 x 4 matrix of control points

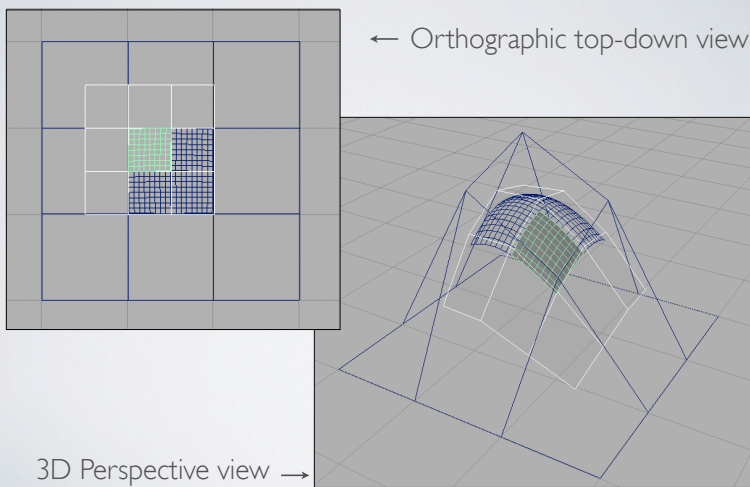


Regular B-Spline Subdivision



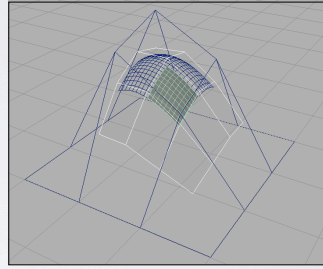
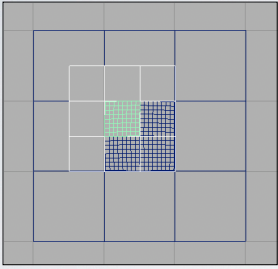
3D Perspective view →

Regular B-Spline Subdivision



3D Perspective view →

Regular B-Spline Subdivision

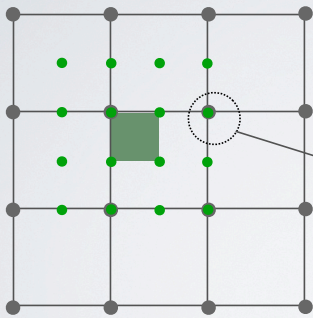


$$\mathbf{x}(u, v) = [1, u, u^2, u^3] \beta_B \mathbf{P} \beta_B^T [1, v, v^2, v^3]^T$$

$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

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Regular B-Spline Subdivision



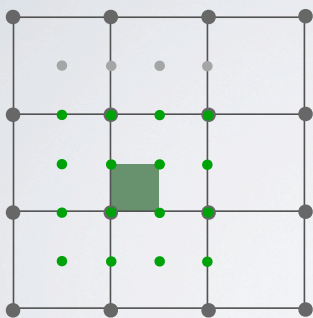
$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

In this parametric view these knot points are collocated.

The 3D control points are not.

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Regular B-Spline Subdivision

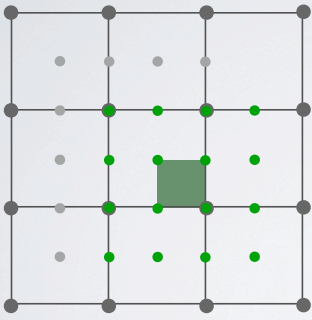


$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

$$\mathbf{P}_{12} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B2}^T$$

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Regular B-Spline Subdivision

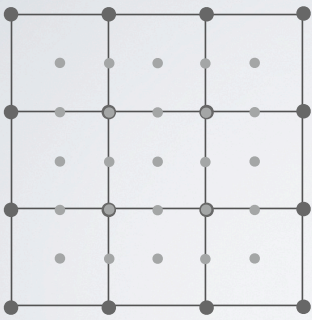


$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

$$\mathbf{P}_{12} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B2}^T$$

$$\mathbf{P}_{22} = \mathbf{H}_{B2} \mathbf{P} \mathbf{H}_{B2}^T$$

Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B1}^T$$

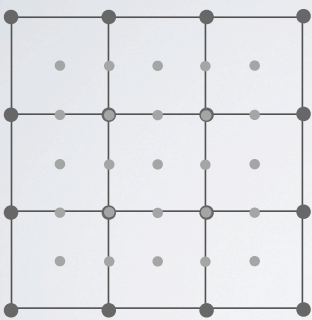
$$\mathbf{P}_{12} = \mathbf{H}_{B1} \mathbf{P} \mathbf{H}_{B2}^T$$

$$\mathbf{P}_{22} = \mathbf{H}_{B2} \mathbf{P} \mathbf{H}_{B2}^T$$

$$\mathbf{P}_{21} = \mathbf{H}_{B2} \mathbf{P} \mathbf{H}_{B1}^T$$

$$\mathbf{H}_{B1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \end{bmatrix} \quad \mathbf{H}_{B2} = \begin{bmatrix} \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Regular B-Spline Subdivision



$$\mathbf{P}^{i+1} = \mathbf{H} \mathbf{P}^i$$

Length 25 vector of fine CPs

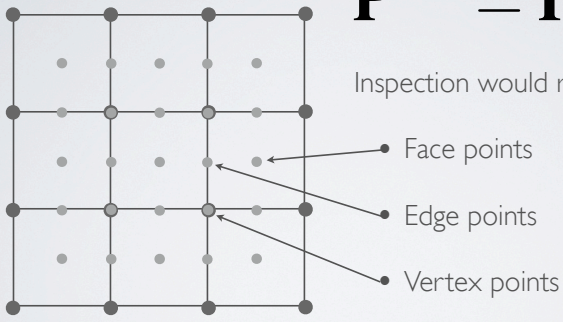
Length 16 vector of coarse CPs

25 × 16 subdivision matrix

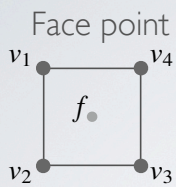
Regular B-Spline Subdivision

$$\mathbf{P}^{i+1} = \mathbf{H} \mathbf{P}^i$$

Inspection would reveal a pattern

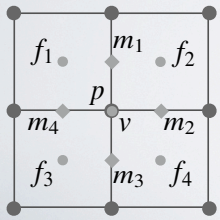
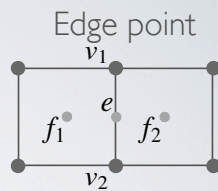


Regular B-Spline Subdivision



$$f = \frac{v_1 + v_2 + v_3 + v_4}{4}$$

$$e = \frac{v_1 + v_2 + f_1 + f_2}{4}$$



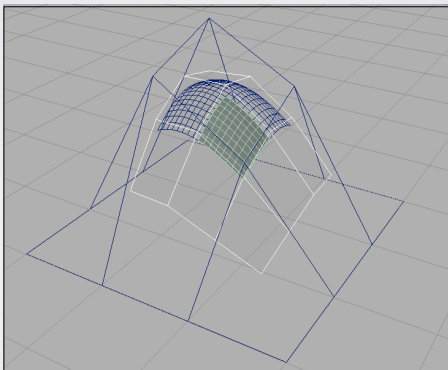
Vertex point

$$v = \frac{f_1 + f_2 + f_3 + f_4 + 2(m_1 + m_2 + m_3 + m_4) + 4p}{16}$$

m midpoint of edge, not "edge point"
 p old "vertex point"

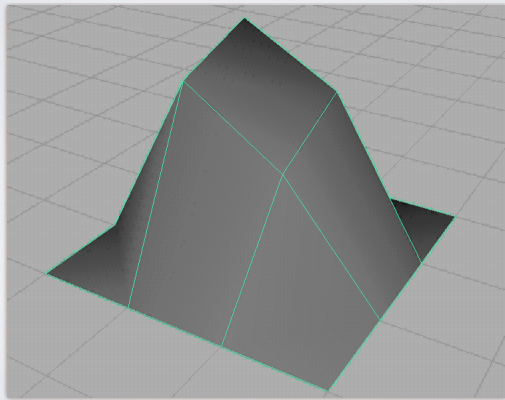
Regular B-Spline Subdivision

Recall that control mesh approaches surface



Regular B-Spline Subdivision

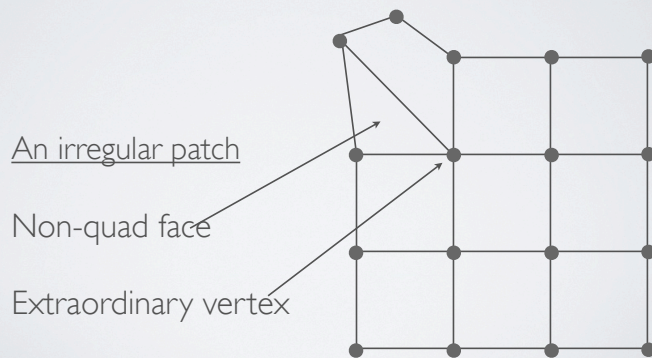
Limit of subdivision is the surface



Irregular B-Spline Subdivision

Catmull-Clark Subdivision

- Generalizes regular B-Spline subdivision



Irregular B-Spline Subdivision

Catmull-Clark Subdivision

- Generalizes regular B-Spline subdivision
- Rules reduce to regular for ordinary vertices/faces

f = average of surrounding vertices

$$e = \frac{f_1 + f_2 + v_1 + v_2}{4}$$

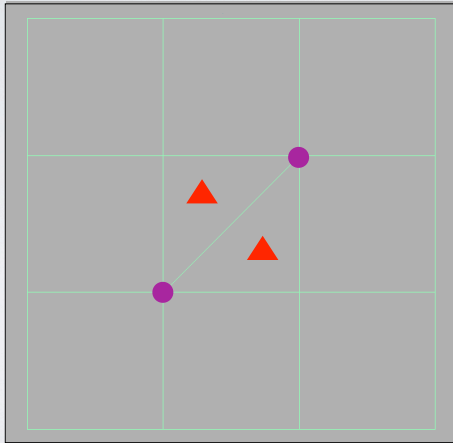
$$v = \frac{\bar{f}}{n} + \frac{2\bar{m}}{n} + \frac{p(n-3)}{n}$$

\bar{m} = average of adjacent midpoints

\bar{f} = average of adjacent face points

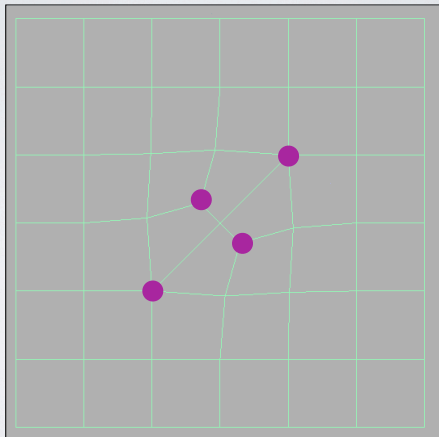
n = valence of vertex

Catmull-Clark Subdivision



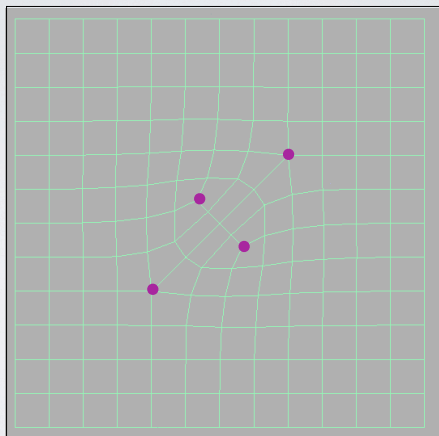
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Catmull-Clark Subdivision



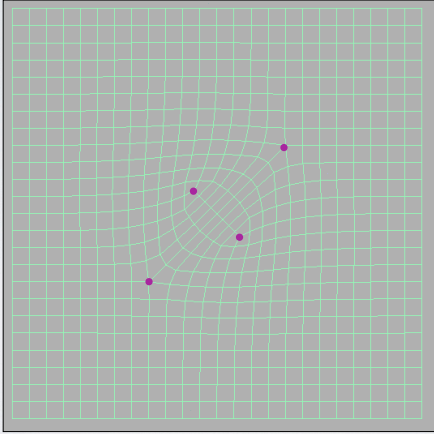
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Catmull-Clark Subdivision



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Catmull-Clark Subdivision



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Continuity of Catmull-Clark

In "ordinary" regions

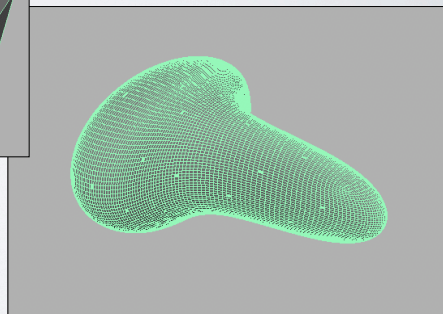
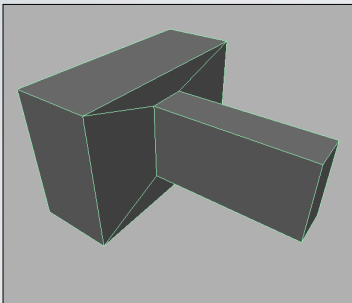
- Surface is fully C^2 everywhere except extraordinary points
- Fast evaluation by matrix exponentiation
 - See "Exact Evaluation Of Catmull-Clark Subdivision Surfaces At Arbitrary Parameter Values" by Jos Stam, SIGGRAPH 1998.

At extraordinary points

- Surface is at least C^1
- Curvature is Lipschitz continuous at extraordinary points

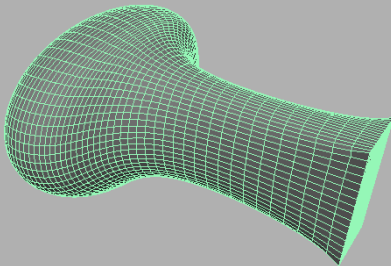
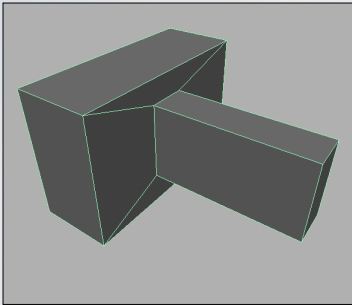
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Catmull-Clark Subdivision

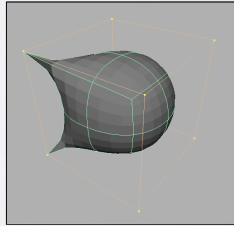
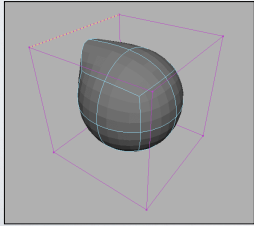
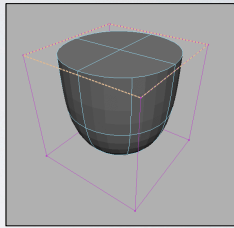
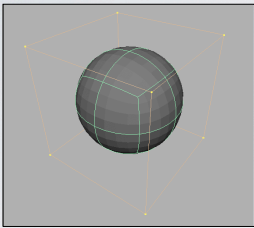


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Catmull-Clark Subdivision



Catmull-Clark Subdivision



Catmull-Clark Subdivision

