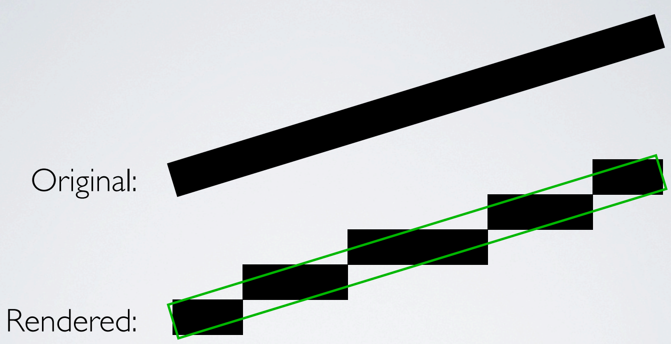
ANTIALIASING

Based on slides by Kurt Akeley

ALIASING

Aliases are low frequencies in a rendered image that are due to higher frequencies in the original image.

JAGGIES



CONVOLUTION THEOREM

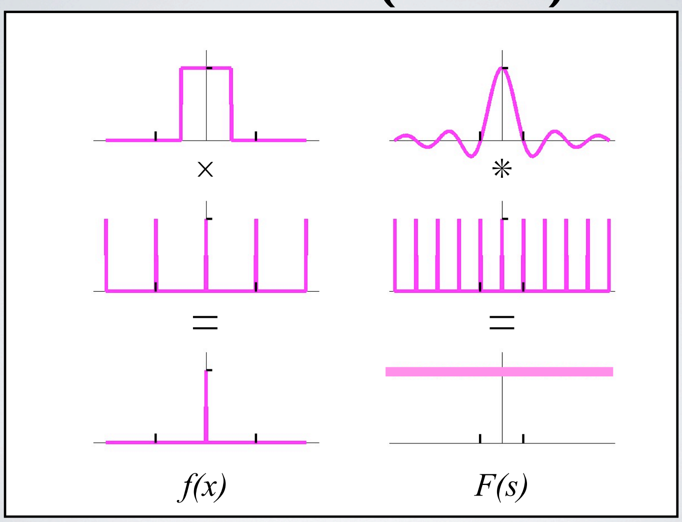
Let $\overline{f}$ and $\overline{g}$ be the transforms of f and g. Then

$$\overline{f * g} = \overline{f} \cdot \overline{g} \qquad f * g = \overline{\overline{f} \cdot \overline{g}}$$

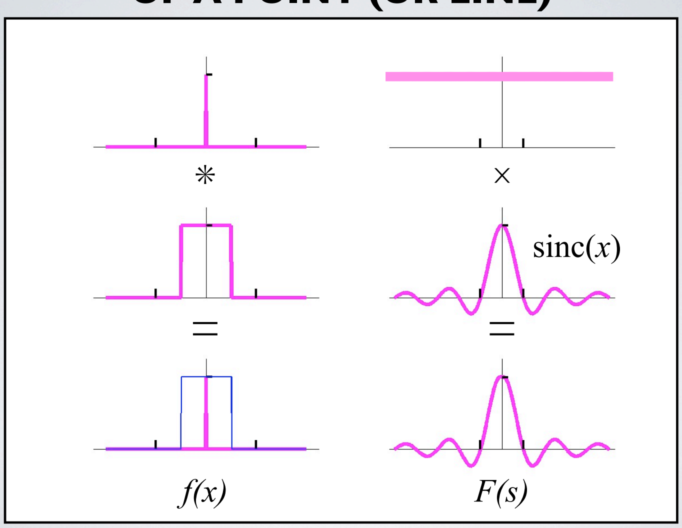
$$\overline{f \cdot g} = \overline{f} * \overline{g} \qquad f \cdot g = \overline{\overline{f} * \overline{g}}$$

Something difficult to do in one domain (e.g., convolution) may be easy to do in the other (e.g., multiplication)

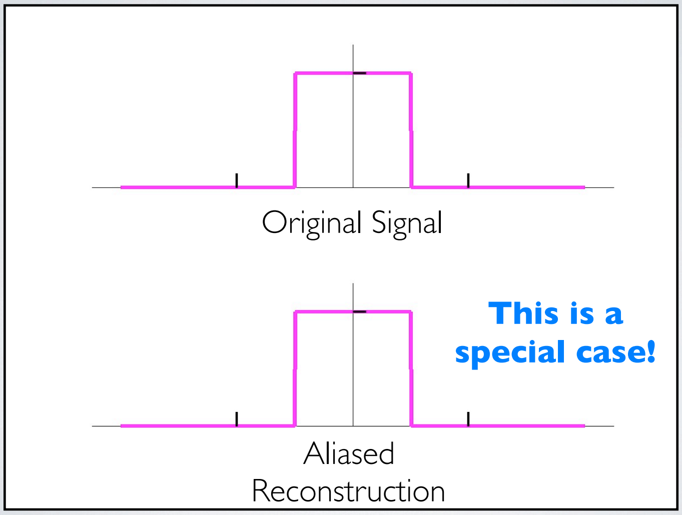
ALIASED SAMPLING OF A POINT (OR LINE)



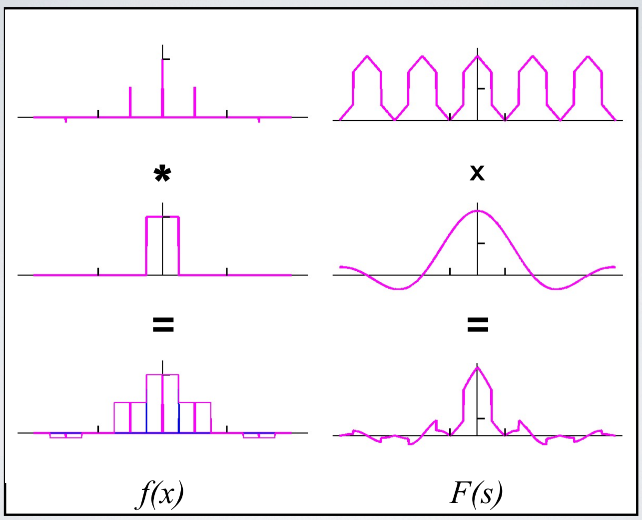
ALIASED RECONSTRUCTION OF A POINT (OR LINE)



RECONSTRUCTION ERROR (ACTUALLY NONE!)



RECONSTRUCTION ERROR



SAMPLING THEORY

Fourier theory explains jaggies as aliasing.

For correct reconstruction:

- Signal must be band-limited
- Sampling must be at or above Nyquist rate
- Reconstruction must be done with a sinc function

All of these are difficult or impossible in the general case. Let's see why …

WHY BAND-LIMITING IS DIFFICULT

Band-limiting changes (spreads) geometry

- Finite spectrum → infinite spatial extent

Interferes with occlusion calculations

Leaves visible seams between adjacent triangles

Can't band-limit the final image

- There ***is*** no final image, there are only samples
- If the sampling is aliased, there is no recovery

Nyquist-rate sampling requires band-limiting

WHY IDEAL RECONSTRUCTION IS DIFFICULT

In theory:

- Required sinc function has
  - Negative lobes (displays can't produce negative light)
  - Infinite extent (cannot be implemented)

In practice:

- Reconstruction is done by a combination of
  - Physical display characteristics (CRT, LCD, …)
  - The optics of the human eye
- Mathematical reconstruction (as is done, for example, in high-end audio equipment) is not practical at video rates.

TWO ANTIALIASING APPROACHES ARE PRACTICED

Pre-filtering

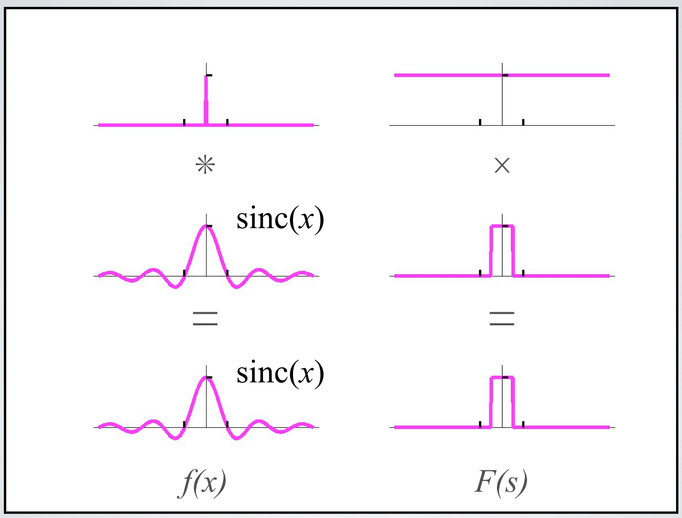
- Band-limit primitives prior to sampling
- OpenGL 'smooth' antialiasing

Increased sample rate

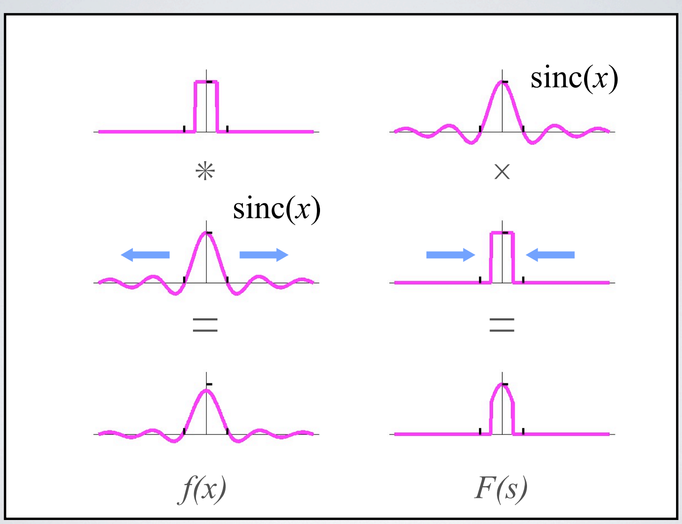
- Multisampling, super-sampling, distributed raytracing
- OpenGL 'multisample' antialiasing
- Effectively sampling above the Nyquist limit

PRE-FILTER ANTIALIASING

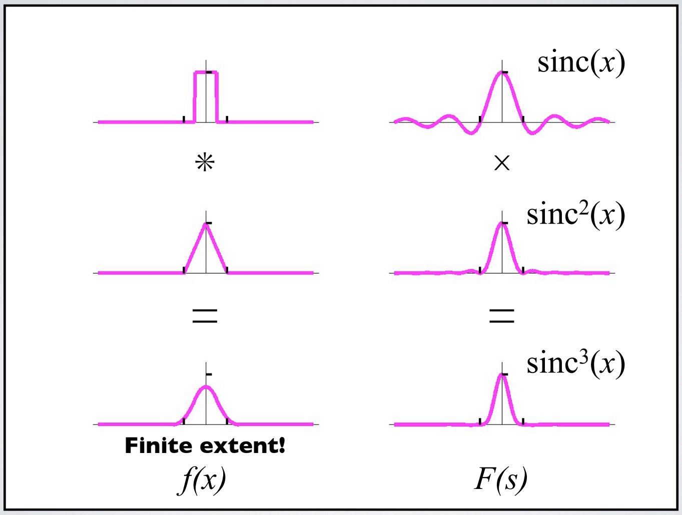
IDEAL POINT (OR LINE CROSS SECTION)



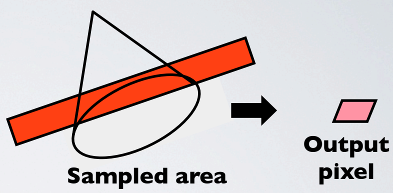
BAND-LIMITED UNIT DISK



ALMOST-BAND-LIMITED UNIT DISK



EQUIVALENCES



These are equivalent:

- Low-pass filtering
- Band-limiting
- (Ideal) reconstruction

These are equivalent:

- Point sampling of band-limited geometry
- Weighted area sampling of full-spectrum geometry

PRE-FILTER PROS AND CONS

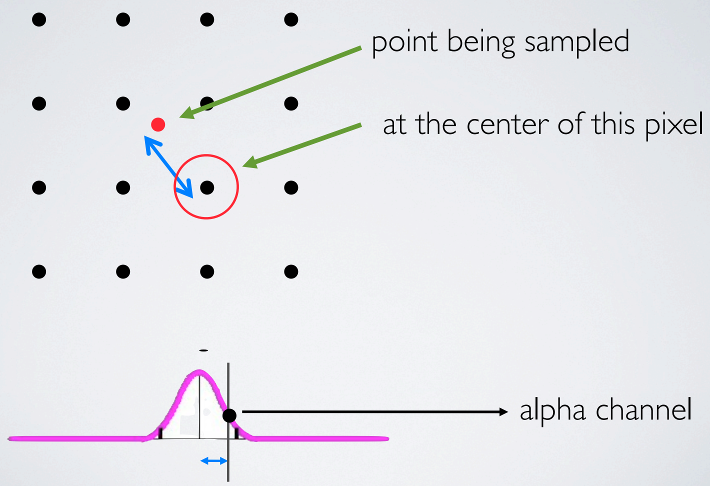
Pros

- Almost eliminates aliasing due to undersampling
- Allows pre-computation of expensive filters
- Great image quality (with some caveats!)

Cons

- Can't handle interactions of primitives with area
- Reconstruction is still a problem

PRE-FILTER POINT RASTERIZATION



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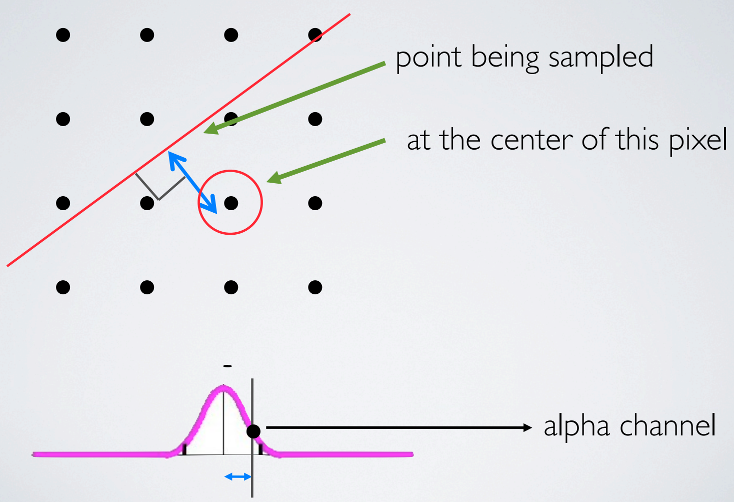
PRE-FILTER LINE RASTERIZATION



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DEMO

http://people.csail.mit.edu/ericchan/articles/prefilter/ 21

DEMO

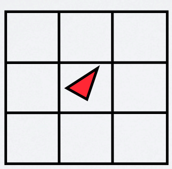
http://people.csail.mit.edu/ericchan/articles/prefilter/

TRIANGLE PRE-FILTERING IS DIFFICULT

Three arbitrary vertex locations define a huge parameter space

Edges can be treated independently

- But this is a poor approximation near vertexes, where two or three edges affect the filtering
- And triangles can be arbitrarily small

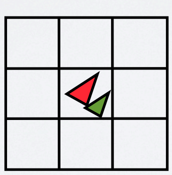


PRACTICAL PRE-FILTERED IMAGE ASSEMBLY

Solution: Weight fragments' contributions according to the area they cover

$$C'_{pixel} = (1-A_{frag})C_{pixel} + A_{frag}\, C_{frag}$$

(Similar to alpha blending)



MULTISAMPLE ANTIALIASING (FOR RAYTRACING)

RECAP

Two approaches to antialiasing

- Pre-filter
- Over-sample: increase sample rate

Pre-filter

- Works well for points and lines, not for triangles
  - Can't reasonably represent pre-filtered triangles
  - Can't handle occlusion
- Can effectively eliminate aliasing with a moderately-large filter window

Over-sample

- Works well for all primitives
  - Though less well than pre-filtering for points and lines
- Cannot eliminate aliasing!
  - Let's see why …

ALIASING OF FUNDAMENTALS AND HARMONICS

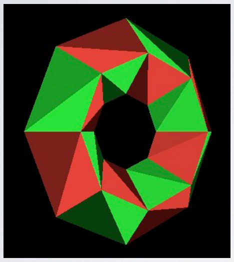
Imperfect analogy, but useful pedagogical tool:

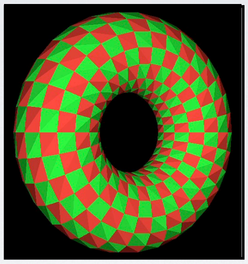
- Fundamental: spatial density of edges
- Harmonics: spectrum introduced by a single edge

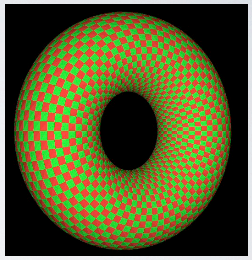
Over-sampling cannot overcome increased fundamental frequency

But over-sampling, followed by a resampling at a lower rate, can overcome harmonics due to a single edge

EXAMPLES OF LEVELS OF DETAIL







Low LOD **Medium LOD** **High LOD**

PRACTICAL SOLUTION

Over-sample to reduce jaggies due to edges

- Optimize for the case of a single edge

Use other approaches to limit edge rates

- Band-limit fundamental geometric frequencies
- Band-limit image fundamentals and harmonics

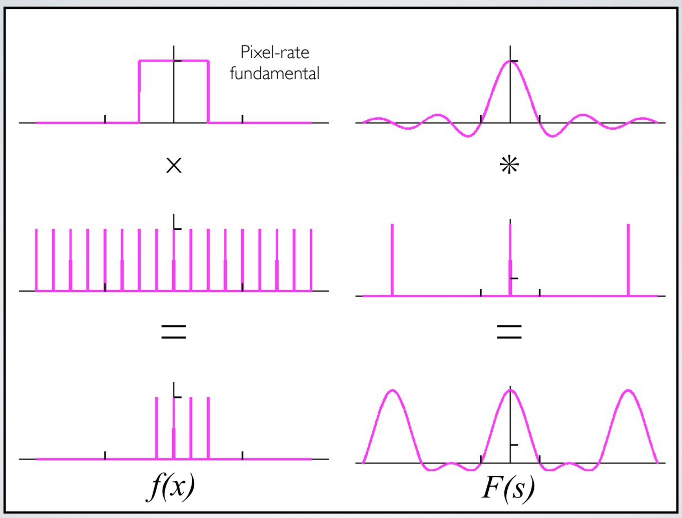
SUPERSAMPLING

SUPERSAMPLING

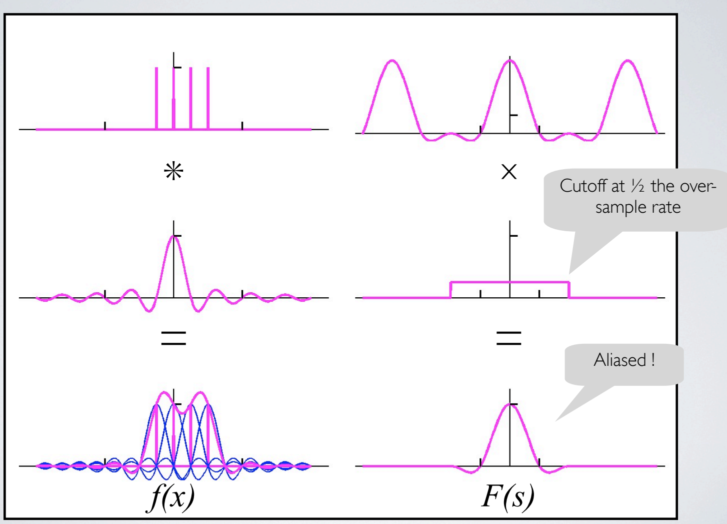
Supersampling algorithm:

1. Over-sample, e.g., at 4x the pixel rate
2. Reconstruct at the over-sampled rate
3. Band-limit to match the pixel rate
4. Resample at the pixel rate to yield pixel values
5. Reconstruct to display

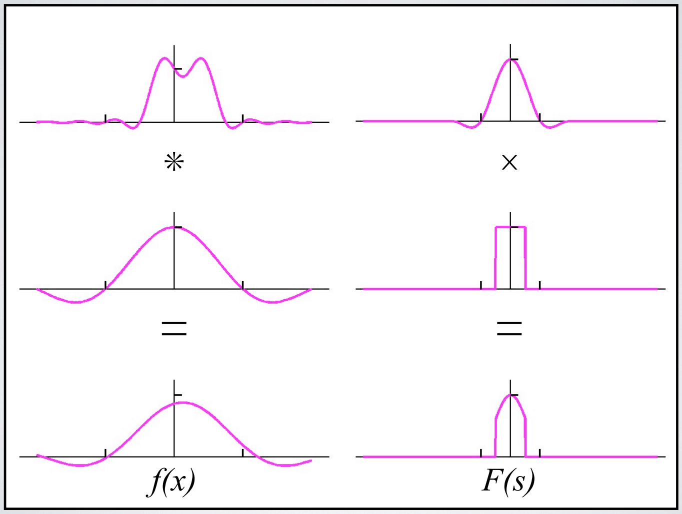
SAMPLE AT 4X PIXEL RATE



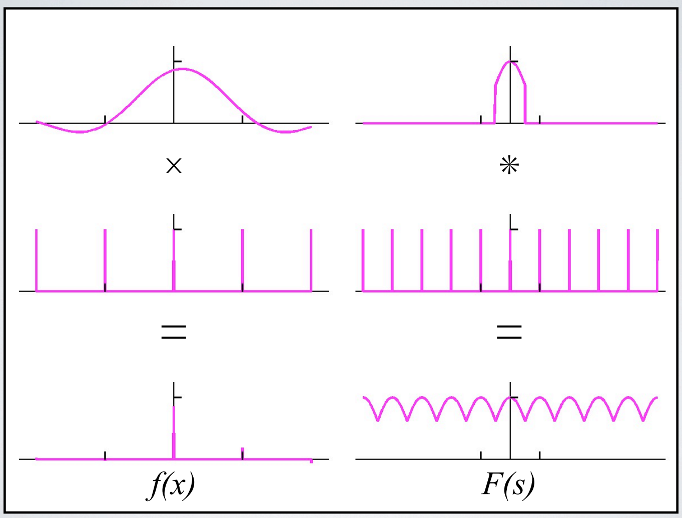
RECONSTRUCT 4X SAMPLES



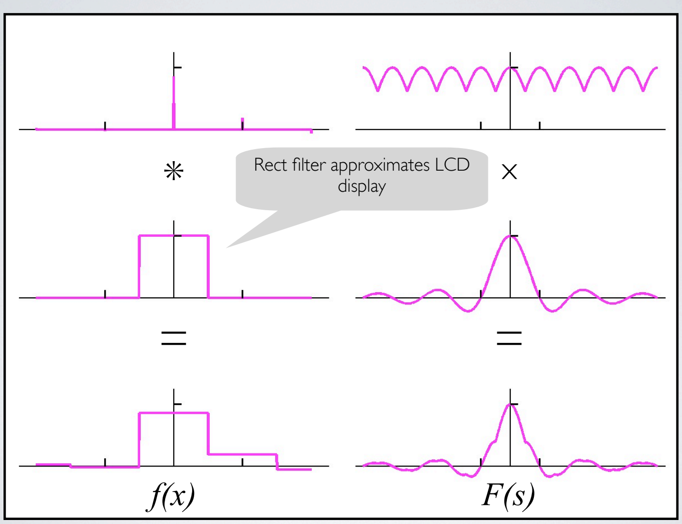
BAND-LIMIT 4X RECONSTRUCTION TO PIXEL-RATE



RESAMPLE AT PIXEL RATE



RECONSTRUCT PIXEL SAMPLES



OPTIMIZATIONS

The over-sample reconstruction convolution and the band-limiting convolution steps can be combined:

- Convolution is associative
  - $(f * g) * h = f * (g * h)$
- And g and h are constant
  - $f * (g * h) = f * filter$

The ***filter*** convolution can be reduced to a simple weighted sum of sample values:

- The result is sampled at pixel rate
- So only values at pixel centers are needed
- These are weighted sums of the 4x samples

OPTIMIZED SUPERSAMPLING

Do once

1. Compute ***filter*** by convolving the over-sample reconstruction and band-limiting filters
2. Compute ***filter weights*** for a single pixel sample by sampling the ***filter*** waveform

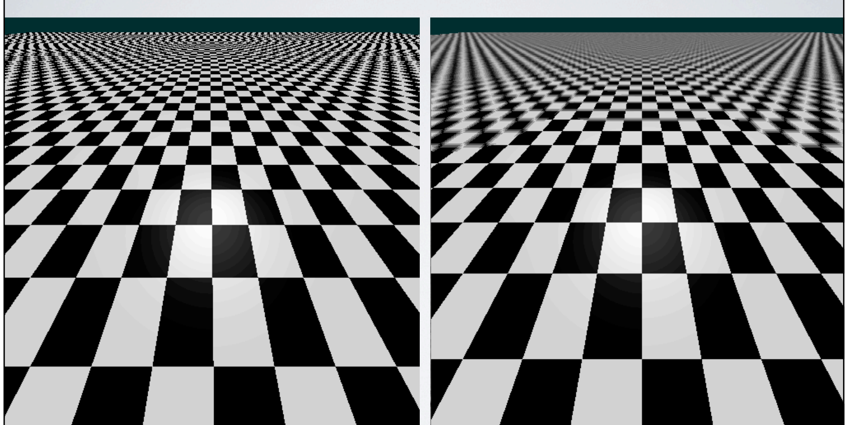
During rendering:

1. Over-sample
2. Filter to compute pixel values from sample values using the ***filter weights***

During display

- Reconstruct

OPTIMIZED SUPERSAMPLING



SAMPLE PATTERNS

SAMPLE PATTERNS

Pattern categories

- Regular grid
- Random samples
- Pseudo-random (jittered grid)

Pattern variation

- Temporal
- Spatial

REGULAR-GRID SAMPLE PATTERN

Implementation

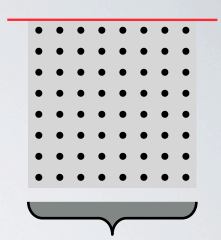
- Regular sample spacing simplifies attribute evaluation
  - Mask, color, and depth
- Large sample count is expensive

Edge optimization

- Poor, especially for screen-aligned edges

Image quality

- Good for large numbers of samples



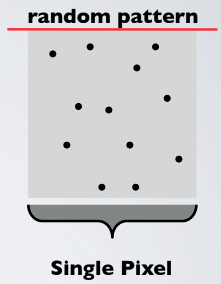
RANDOM-SAMPLE PATTERN

Implementation
- Irregular sample spacing complicates attribute evaluation
- Requires moderate sample size to avoid noise

Edge optimization
- Good, though less optimal for screen-aligned edges

Image quality
- Excellent for moderate numbers of samples

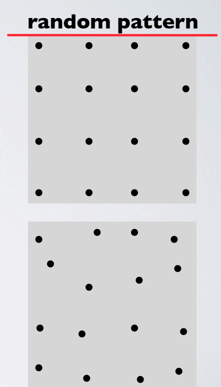
Temporal issues
- Must assign the same pattern to a given pixel in each frame



JITTERED-GRID PATTERN

Implementation
- Best trade-off of sample number, noise level, and anti-aliasing

Temporal issues
- Must assign the same pattern to a given pixel in each frame



SUMMARY

Supersampling does ***not*** eliminate aliasing
- It can reduce jaggies
- It must be used with other pre-filtering to eliminate aliasing

Multisampling
- Is supersampling with pixel-rate shading
  - Optimized for efficiency and performance
- Is a full-scene antialiasing technique
  - Works for all rendering primitives
- Handles occlusions correctly
- Can be used in conjunction with pre-filter antialiasing
  - To optimize point and line quality

SUMMARY

Two approaches to antialiasing are practiced

- Pre-filtering
- Multisampling

Pre-filter antialiasing

- Works well for non-area primitives (points, lines)
- Works poorly for area primitives (triangles, especially in 3-D)

Multisampling antialiasing

- Works well for all primitives
- Supersampling alone does not help--need to filter down to pixel resolution

FILTERING AND HUMAN PERCEPTION

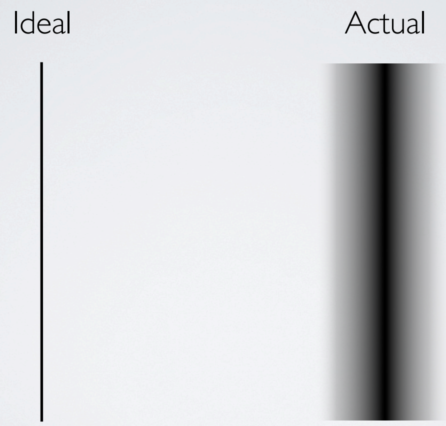
RESOLUTION OF THE HUMAN EYE

Eye's resolution is not evenly distributed

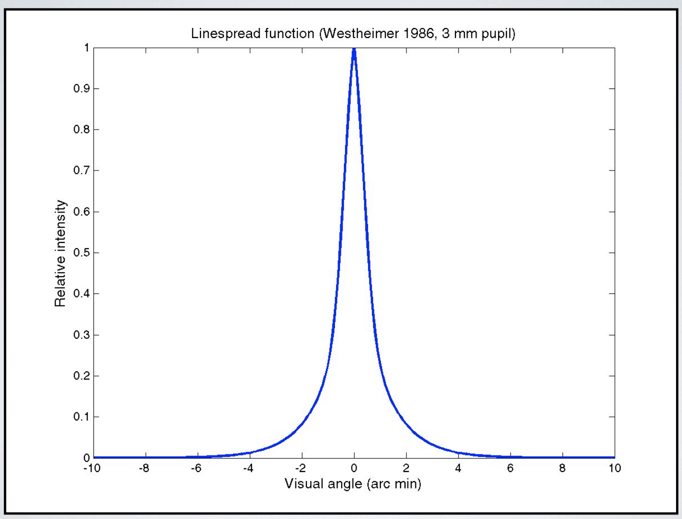
- Foveal resolution is ~20x peripheral
- Systems can track direction of view, draw high-resolution inset
- Flicker sensitivity is higher in periphery

Human visual system is well engineered …

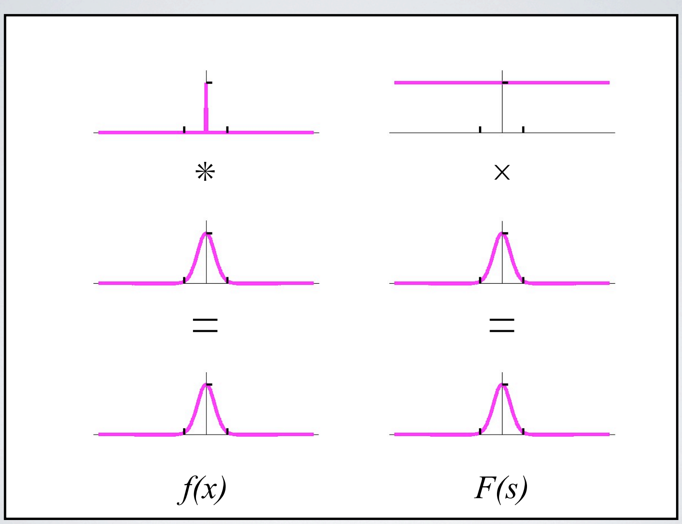
IMPERFECT OPTICS - LINESPREAD



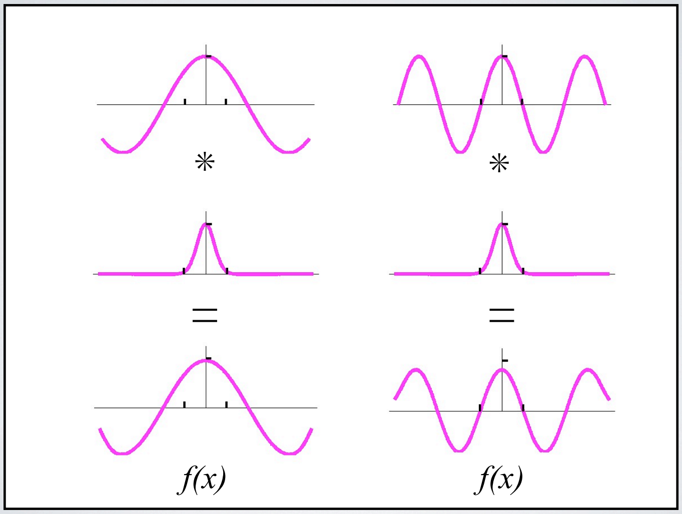
LINESPREAD FUNCTION



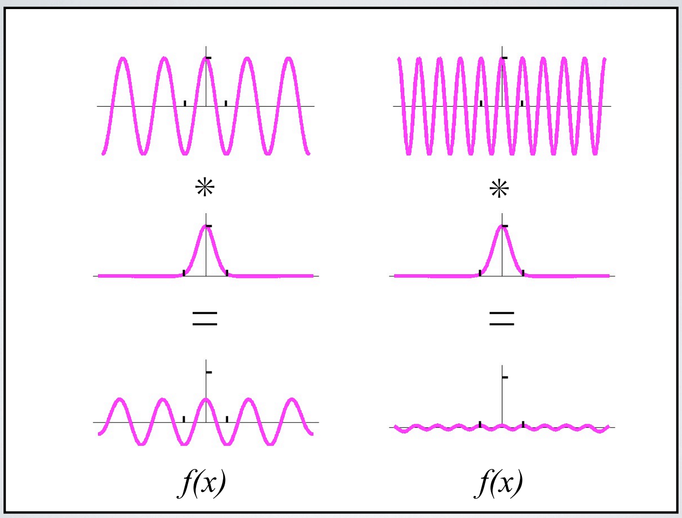
FILTERING



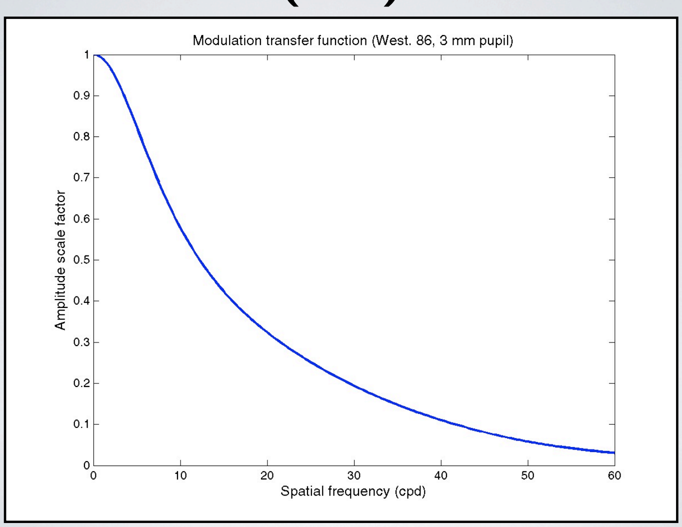
FREQUENCIES ARE SELECTIVELY ATTENUATED



SELECTIVE ATTENUATION (CONT.)



MODULATION TRANSFER FUNCTION (MTF)



OPTICAL "IMPERFECTIONS" IMPROVE ACUITY

Image is pre-filtered

- Cutoff is approximately 60 cpd
- Cutoff is gradual – no ringing

Aliasing is avoided

- Foveal cone density is 120 / degree
- Cutoff matches retinal Nyquist limit

Vernier acuity is improved

- Foveal resolution is 30 arcsec
- Vernier acuity is 5-10 arcsec
- Linespread blur includes more sensors