CS-184: Computer Graphics

Lecture 9: Sampling and Aliasing

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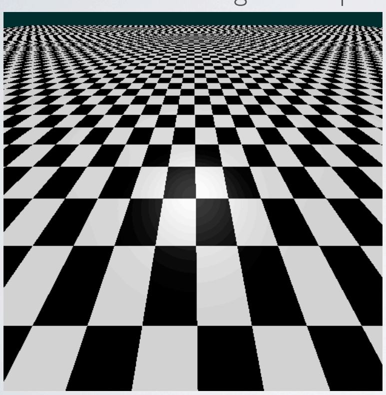
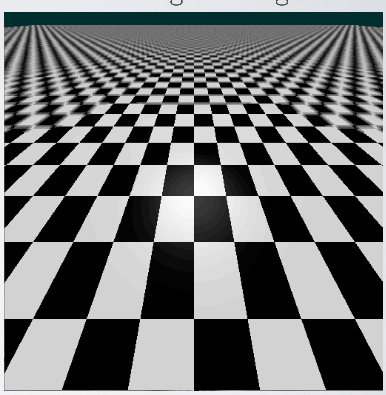
Aliasing

Aliases are low frequencies in a rendered image that are due to higher frequencies in the original image.



aliasing effects

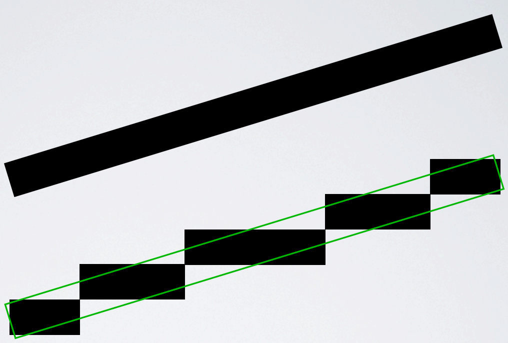


anti-aliased

Jaggies

Original:

Rendered:



Are jaggies due to aliasing? How?

Aliasing (temporal)

http://www.michaelbach.de/ot/mot_wagonWheel/main.swf

Sampling

How to represent a continuous signal digitally?

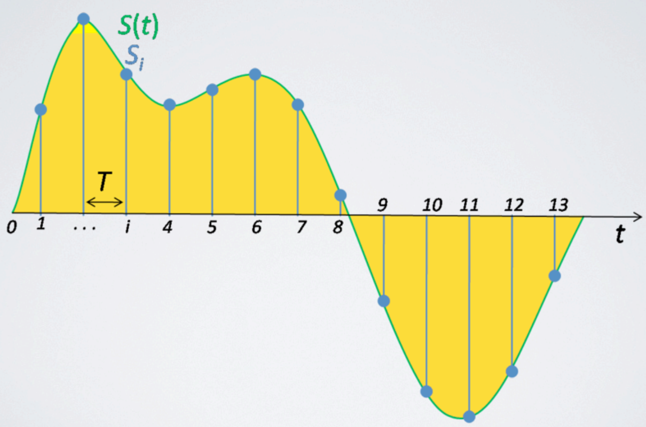
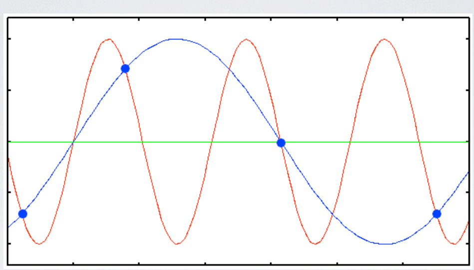


image from Wikipedia

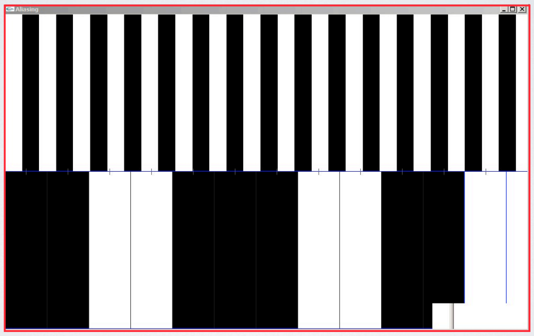
Undersampling



Both frequencies could explain the samples

Aliasing

Aliases are low frequencies in a rendered image that are due to higher frequencies in the original image.



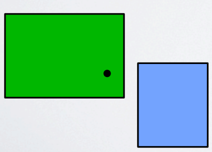
What is a (point) sample?

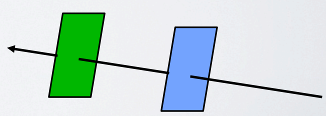
An evaluation

- At an infinitesimal point (2-D)
- Or along a ray (3-D)
- At a particular time (animation/audio)

What is evaluated

- Inclusion (2-D) or intersection (3-D)
- Attributes such as distance and color
- Air pressure (audio)





Questions for this lecture

How can we model/analyze the sampling process?

How can we reconstruct a signal from samples?

When can we do a good job (i.e. avoid aliasing)?

Reference sources

Kurt Akeley's slides

Brian Curless' slides

Marc Levoy's notes

Ronald N. Bracewell, ***The Fourier Transform and its Applications, Second Edition,*** McGraw-Hill, Inc., 1978.

Ground rules

You don't have to be an engineer to get this

- We're looking to develop instinct / understanding
- Not to be able to do the mathematics

We'll make minimal use of equations

- No integral equations
- No complex numbers

Plots will be consistent

- Tick marks at unit distances
- Signal on left, Fourier transform on the right

Dimensions

1-D

- Audio signal (time)
- Generic examples (x)

2-D

- Image (x and y)

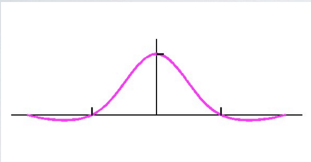
3-D

- Animation (x, y, and time)

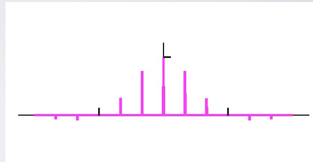
Most examples in this presentation are 1-D

Sampling and Reconstruction

Remove high frequencies before sampling



Continuous Signal



Discrete Samples

Don't introduce spurious high frequencies during reconstruction

Displays are discrete, so why do we need to reconstruct anyway?

- *Resampling*: Scaling up/down, texture mapping, supersampling

Filtering

Filtering is used for both sampling and reconstruction

Sampling: filter high frequencies from continuous signal

- Diffusing filter for cameras or analog audio filter
- Average multiple samples at a higher frequency (*Oversampling*)

Reconstruction: filter samples to interpolate continuous signal

- Reconstruction filters can introduce higher frequencies

Convolution

One of the most common methods for filtering

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)\,du$$

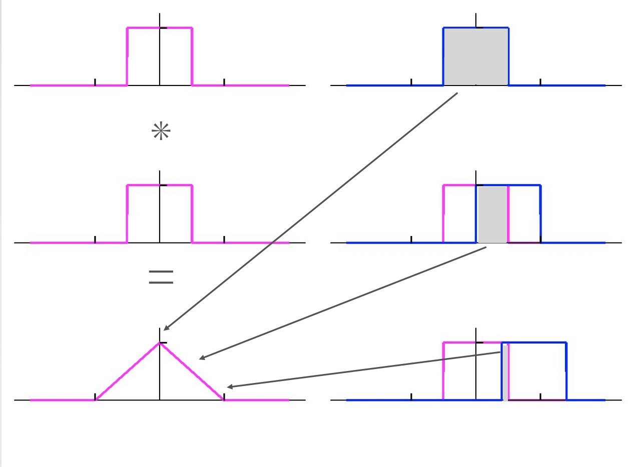
Function f and filter g

$(f * g)(x)$ = shift g by x and take product

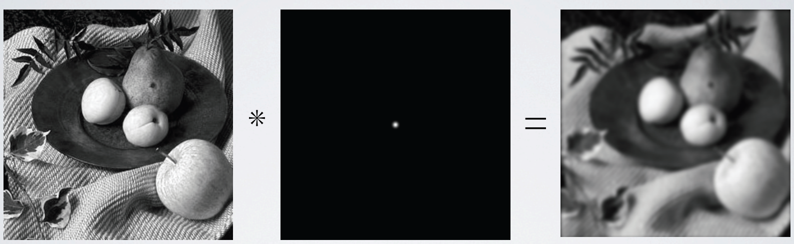
Commutative, associative, distributive

Extends to higher dimensions and discrete functions

Convolution example

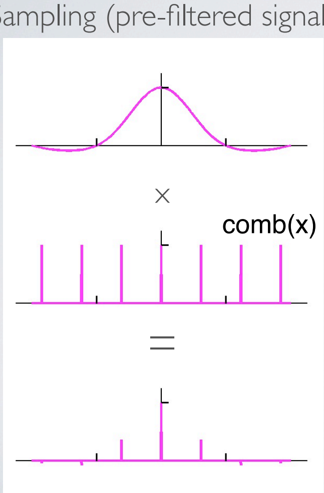


Convolution example (2D)

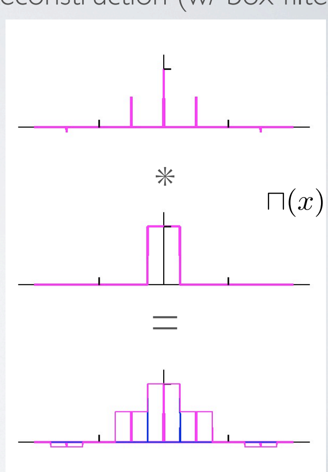


Sampling and Reconstruction

Sampling (pre-filtered signal)



Reconstruction (w/ box filter)



Fourier analysis

The Fourier transform lets us analyze functions in frequency domain

- Natural in conjunction with convolution

Spatial domain $\longrightarrow$ $$F(s)=\int_{-\infty}^{\infty} f(x)e^{-i2\pi sx}dx$$ $\longrightarrow$ Frequency domain

Spatial domain $\longleftarrow$ $$f(x)=\int_{-\infty}^{\infty} F(s)e^{i2\pi sx}ds$$ $\longleftarrow$ Frequency domain

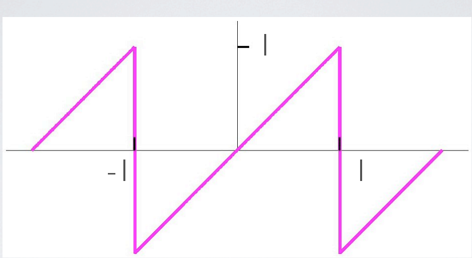
Fourier series

Any periodic function can be exactly represented by a (typically infinite) sum of harmonic sine and cosine functions.

Harmonics are integer multiples of the fundamental frequency

Fourier series example: sawtooth wave

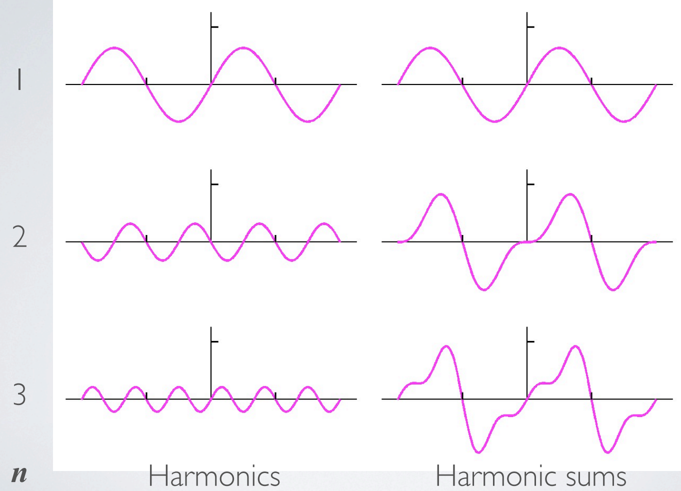
. . .



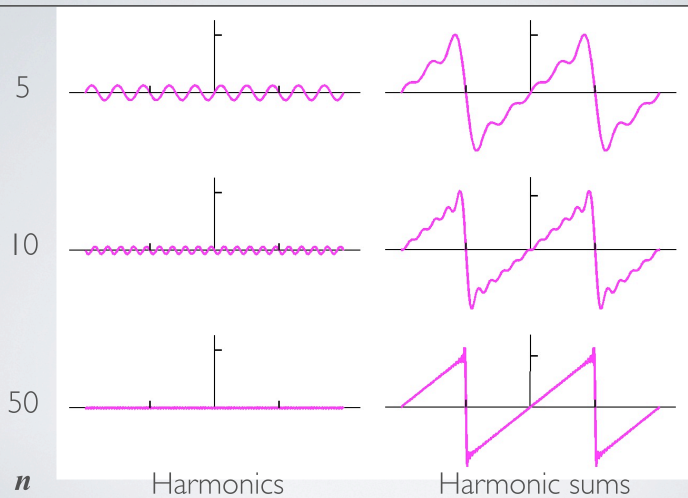
. . .

$$f(x)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(nx\pi)$$

Sawtooth wave summation



Sawtooth wave summation (continued)

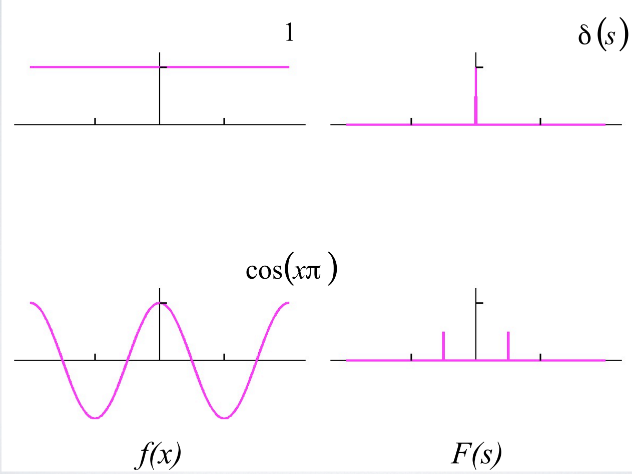


Fourier integral

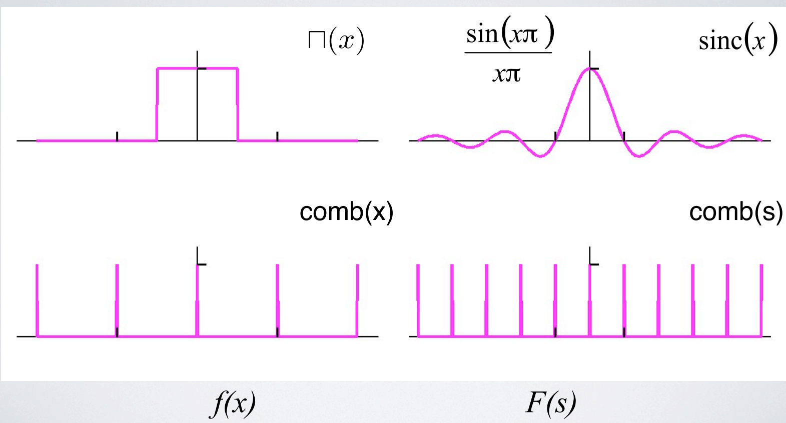
Any function (that matters in graphics) can be exactly represented by an integration of sine and cosine functions.

Continuous, ***not*** harmonic

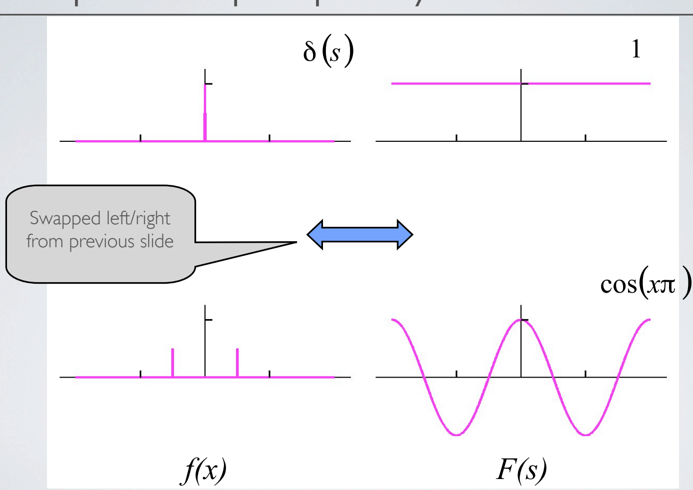
Basic Fourier transform pairs



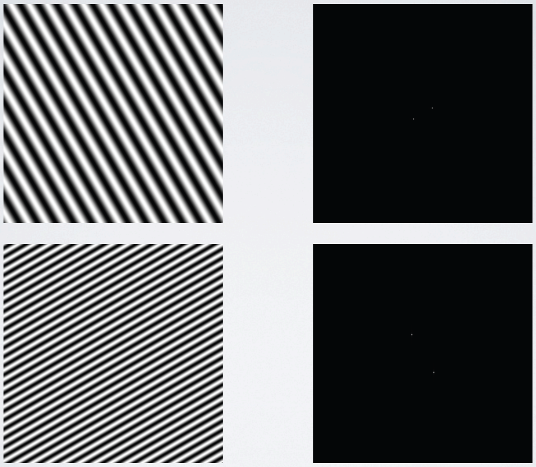
Basic Fourier transform pairs



Reciprocal property

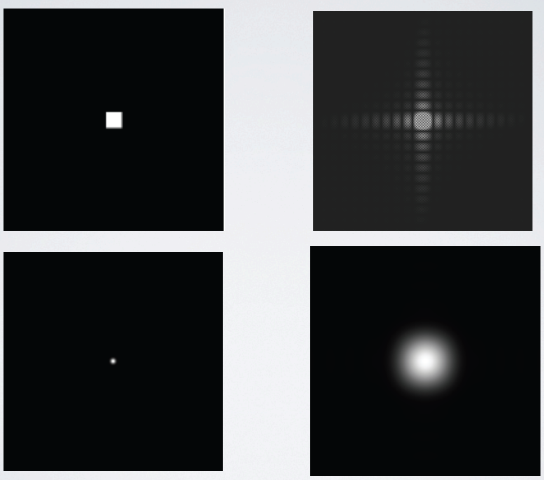


Basic Fourier transform pairs (2D)



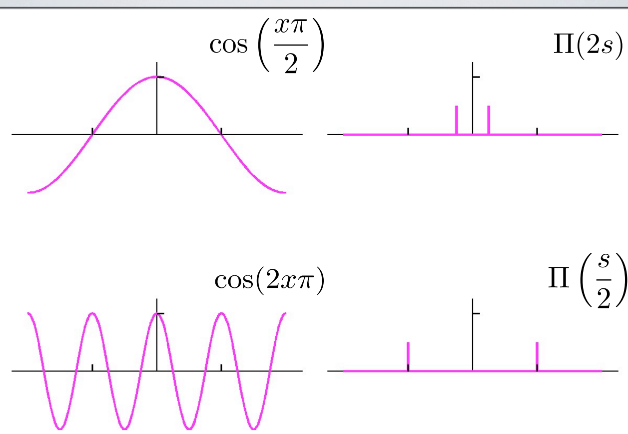
$f(x)$ $F(s)$

Basic Fourier transform pairs (2D)



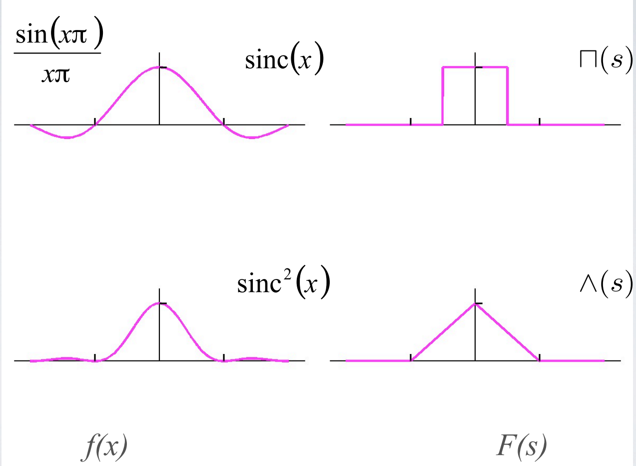
$f(x)$ $F(s)$

Scaling theorem



$f(x)$ $F(s)$

Band-limited transform pairs



Finite / infinite extent

If one member of the transform pair is finite, the other is infinite

- Band-limited → infinite spatial extent
- Finite spatial extent → infinite spectral extent

Convolution theorem

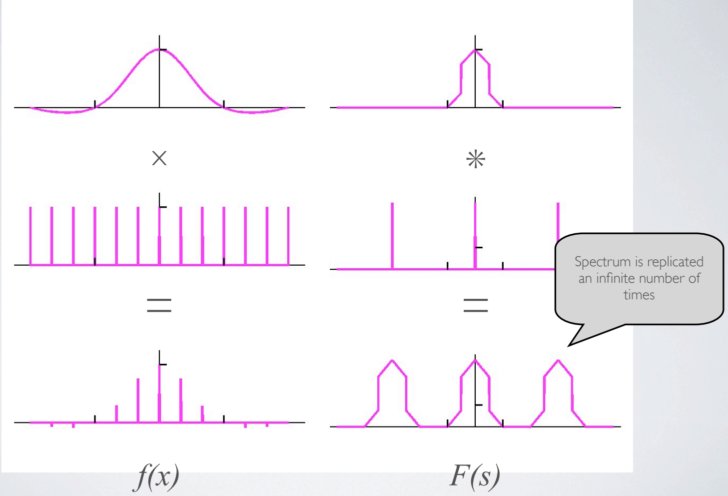
Let $\overline{f}$ and $\overline{g}$ be the transforms of f and g. Then:

$$\overline{f * g} = \overline{f} \cdot \overline{g} \qquad f * g = \overline{\overline{f} \cdot \overline{g}}$$

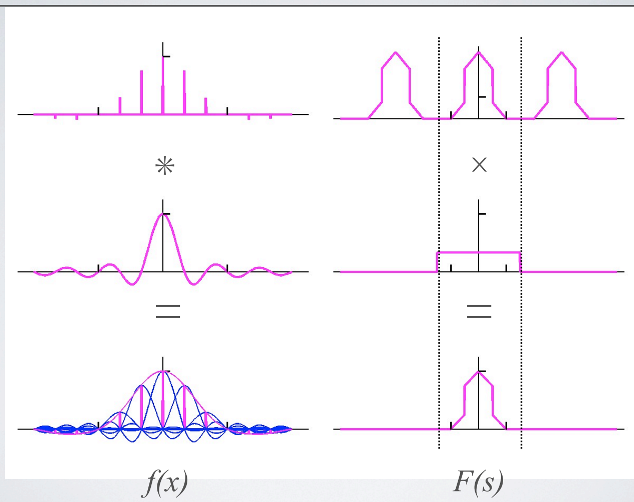
$$\overline{f \cdot g} = \overline{f} * \overline{g} \qquad f \cdot g = \overline{\overline{f} * \overline{g}}$$

Something difficult to do in one domain (e.g., convolution) may be easy to do in the other (e.g., multiplication)

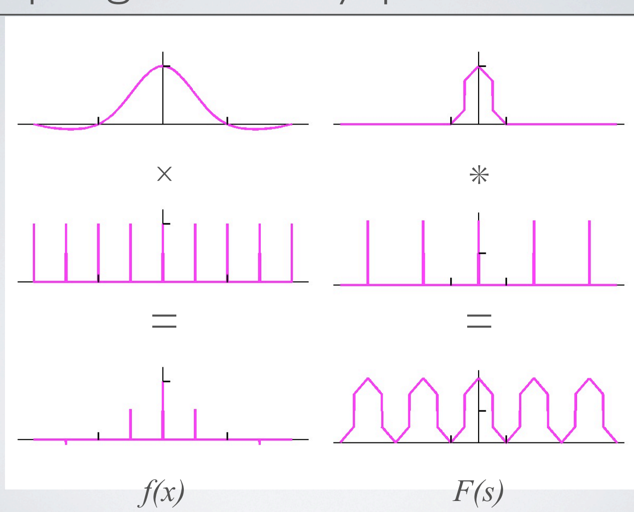
Sampling theory



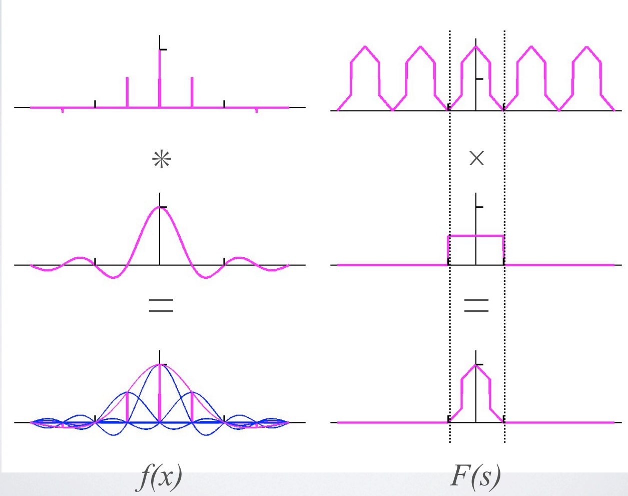
Reconstruction theory



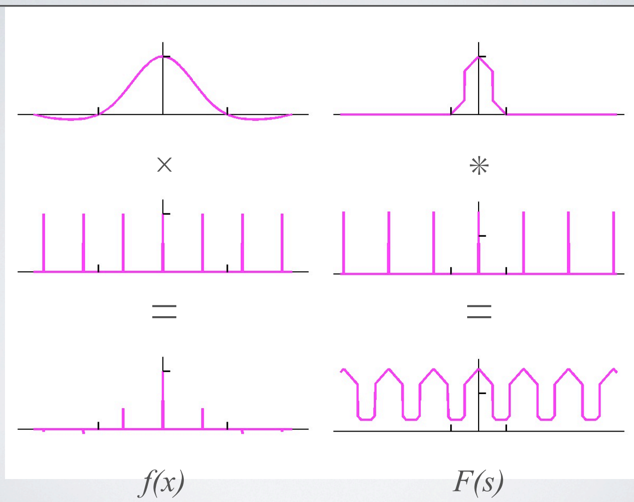
Sampling at the Nyquist rate



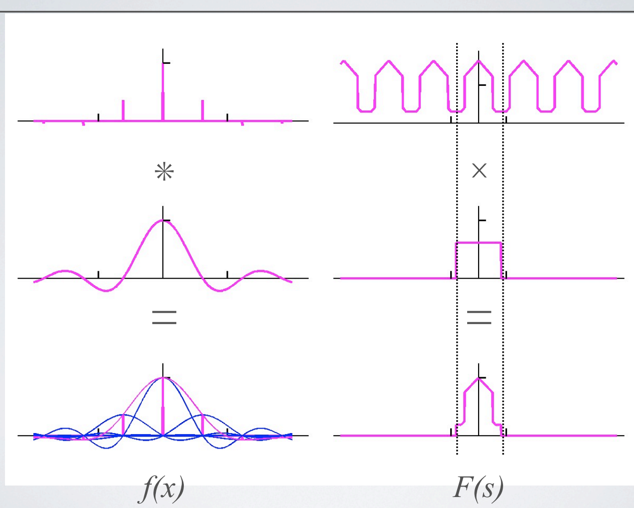
Reconstruction at the Nyquist rate



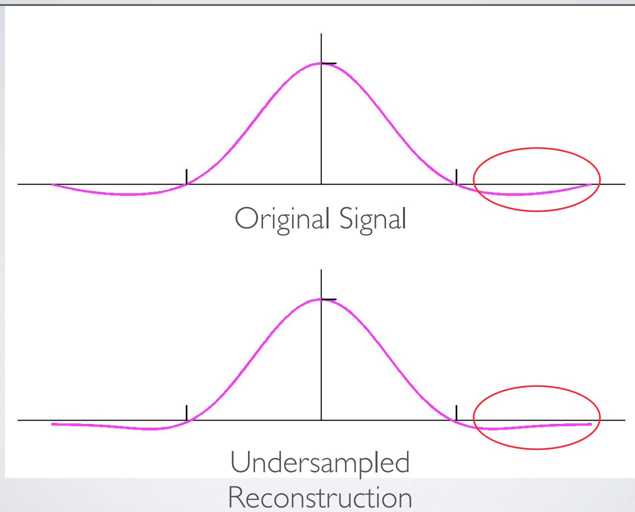
Sampling below the Nyquist rate



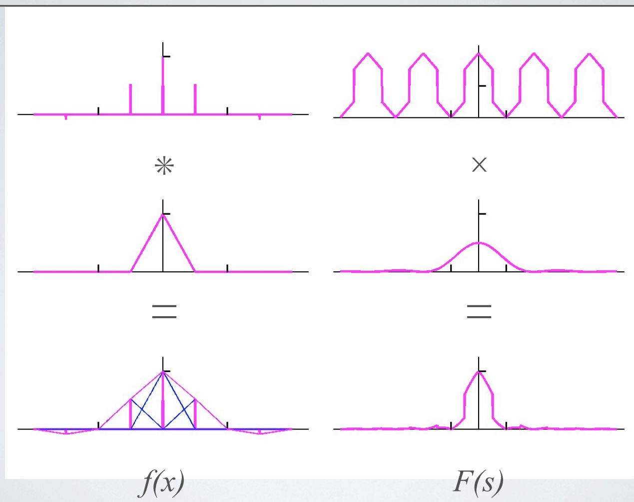
Reconstruction below the Nyquist rate



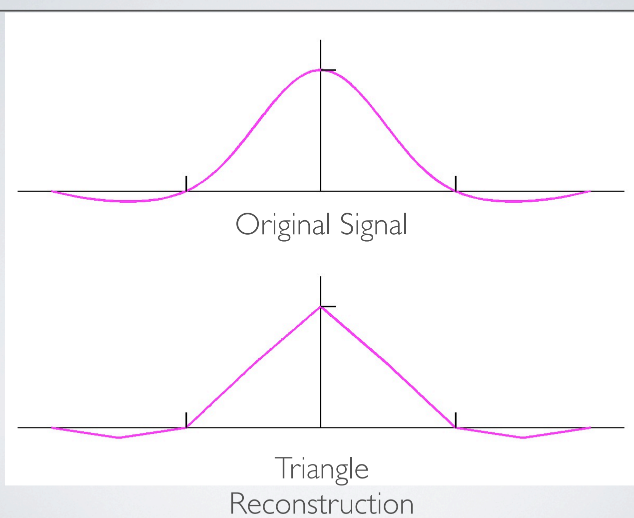
Reconstruction error



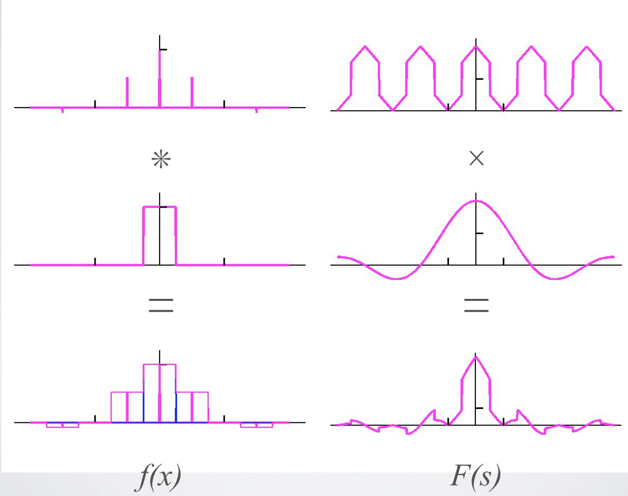
Reconstruction with a triangle function



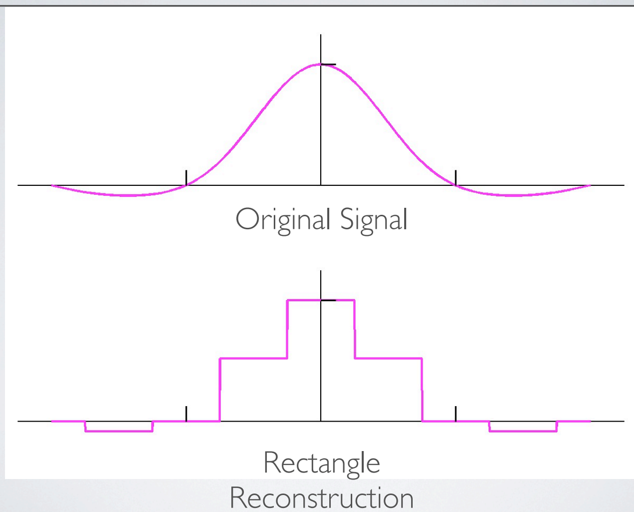
Reconstruction error



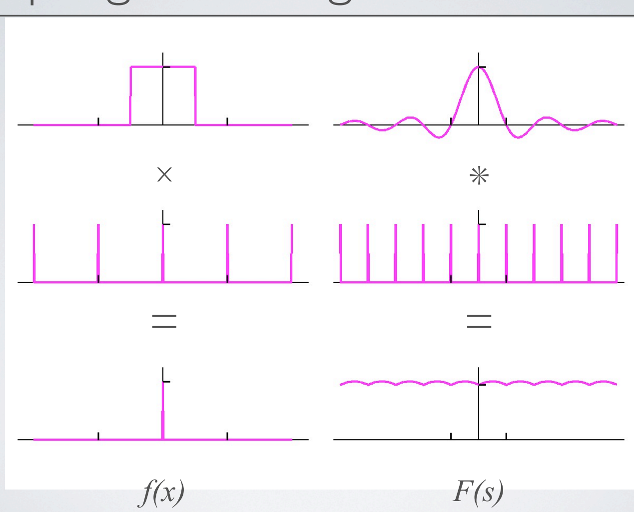
Reconstruction with a rectangle function



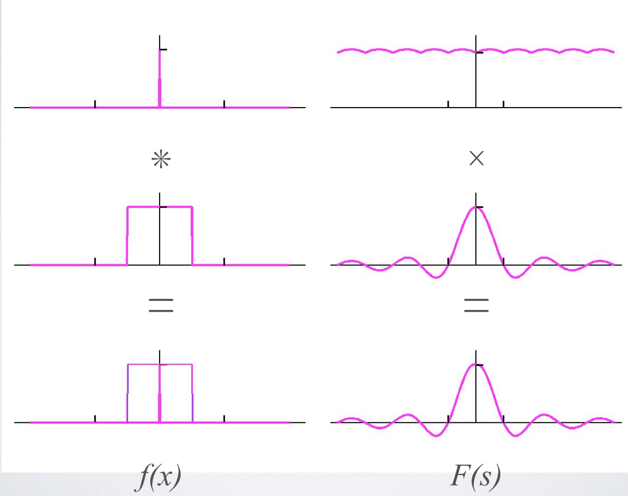
Reconstruction error



Sampling a rectangle



Reconstructing a rectangle (jaggies)



Sampling and reconstruction

Aliasing is caused by

- Sampling below the Nyquist rate,
- Improper reconstruction, or
- Both

We can distinguish between

- Aliasing of fundamentals (demo)
- Aliasing of harmonics (jaggies)

End